CVNG 3004 STRUCTURAL DYNAMICS

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1.0 Introduction

The practice of structural dynamics is essentially the formulation then solution of equations of different types applicable to different conditions. A new student of the topic typically finds themselves a bit overwhelmed and the significance of the different aspects of the topic quickly becomes difficult to grasp. However, it is this significance, which arises when the understanding of the subtopics are synthesized into a whole, that is most required for effective modeling and analysis.

The aim of this introduction is to put the various parts of structural dynamics into perspective – a bird's eye view, which may help the student more easily develop a feel for the relative significance of the various subtopics of structural dynamics.

1.1 The Critical Phenomena of Structural Dynamics

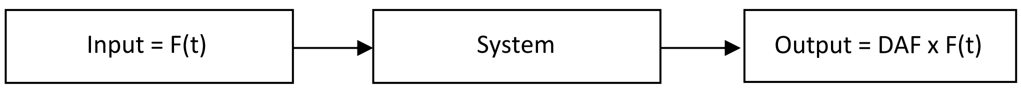

Figure 1

Consider a physical system – a structural system, a mechanical system, an electrical circuit, etc. At some point or points in the system, a time-varying excitation is imposed as an input, and we desire to know how the system will respond, as output. Dynamics, or Vibration Theory, enables us to determine the response and is characterized by just two things – the phenomena are functions of time, and are such that the response is magnified or amplified relative to the input. That is, if the input is F(t), the output is DAF x F(t), where DAF is the dynamic amplification factor. This amplification takes place because the output actually reinforces the input so therefore, dynamics is a feedback phenomenon.

Structural dynamics is the application of Vibration Theory to structures typically encountered in civil engineering, and the amplification is due to the generation of inertia forces that act on the system, as an additional force. Hence in structural dynamics the dependant variable is with respect to the equations of motion associated with Newton's Second Law. Therefore, the variables are displacement, x(t), velocity (i.e dx/dt), and acceleration (i.e. d^2x/dt^2). The input is typically a load or force F(t), and the output is the displacement, x(t), from which the other response quantities of interest are determined (i.e velocity, acceleration, inertia force, etc).

This also applies to Mechanical Engineering but in this arena, the structure is a machine or part of a machine. Vibration Theory also applies to certain aspects of Electrical Engineering, so some of the terminology of structural dynamics originates from this field since the governing equations are the same.

1.2 Structural Dynamics in Civil Engineering Design

Structural dynamics is encountered in civil engineering as the time-varying loads imposed on the various structures common to civil engineering, and the resulting behavior of those structures.

In general, structural design can be defined as the control of phenomena within the structural system by determining the values of its appropriate properties, such that the response of the system is acceptable. Acceptability is in terms of: (1) the safety of the structure against collapse, and (2) the comfort of the user during the service life of the structure. That is, the collapse or ultimate limit state, and the serviceability limit state, respectively.

Regarding the ultimate limit state, the structure is economically safe if the stress demands on the structure are just less than the capacities of the structure. The stress demands are typically in terms of the stress resultants within the system – the moments (M), shears (V), axial forces (P), and torsion (T), collectively called the design actions.

In structures under dynamic loading, the stresses are amplified when its ratio of its mass to stiffness (i.e. M/K) is within a certain range for a given the characteristics of the load. The designer has the following options to control the stress to acceptable levels. (1) Accept the amplified load and proportion the structure so that it has the capacity to resist it. (2) Alter the M/K ratio so that the response is not significantly amplified. (3) Separate the structure from the source of the dynamic load. Lastly, (4) add devices to the structure to absorb the force in those devices instead of within the structural elements.

Regarding the serviceability limit state, the following are generally undesirable. The vibration of a floor or deck as someone or something moves on it, for example, in buildings and bridges. In buildings the movement can be due to a person walking, a group of persons dancing, a crowd jumping as in a stadium, or a machine located on the same floor. The feeling of nausea when a structure sways under wind loading is also undesirable and occurs when the structure is in a particular range of acceleration. If the structure vibrates too much in the event of an impact, say due to the collision of a vehicle, or a blast, this can also be disturbing to the occupants, and cause the belief that the building is unsafe, hence affecting their ability to function normally. To avoid such occurrences, the designer can use the same approaches described above for the ultimate limit state.

The following graphically describes the various types of dynamic loading generally found in civil engineering. For each type of dynamic loading, graphs of a different shape are possible.

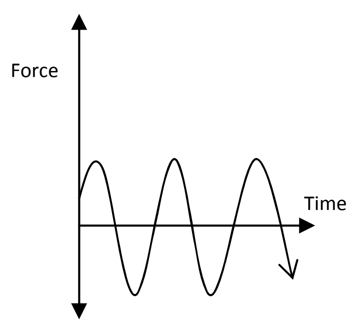

Figure 2a. Periodic Harmonic – Long Duration

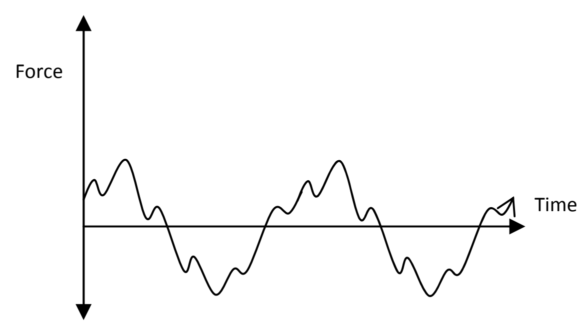

Figure 2b. Periodic Non-Harmonic – Long Duration

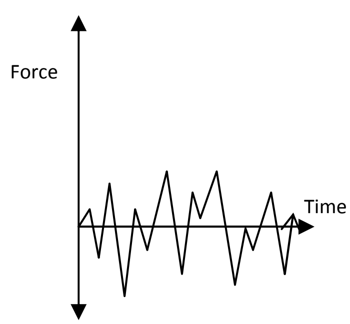

Figure 2c. Non-Periodic – Long Duration

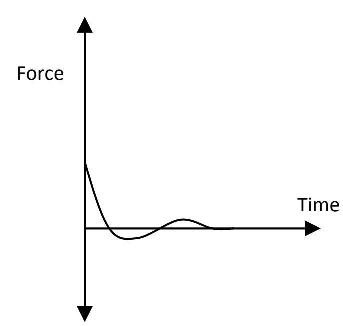

Figure 2d. Non-Periodic – Short Duration (Transient)

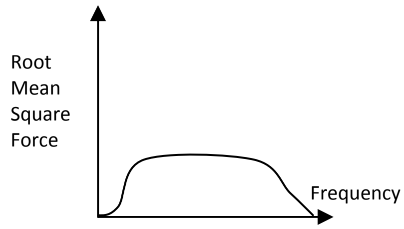

Figure 2e. Spectrum

Examples of these types of dynamic loading are as follows. Periodic harmonic long duration loading (usually just called "harmonic") can be due to machinery on a floor. Periodic non-harmonic long duration loading (usually just called "periodic") can be due to a crowd jumping at a stadium during a game. Non-periodic long duration loading (usually called "non-periodic") can be due to an earthquake recording at a specific point for a specific earthquake. Non-periodic short duration loading (usually called "transient"), can be due to a wave crashing against a sea wall, or a blast, or a collision.

Figures 2a to 2d describe deterministic loadings. That is, the value of the load is exactly known at a point in time, t. However, there are other situations where at any time, t, it is not possible to know the exact value of the load as there is a range of possible values at time, t. Such a phenomenon is called a random process, or a stochastic process, and statistics/probability theory is used in order to quantitatively describe the phenomenon.

An example of a random process is earthquake loading on a structure. We can obtain a recording of, say, the acceleration of a structure during an earthquake, such as shown in Figure 2c. However, for the same structure but at a different point, the graph will not be the same. Likewise, a point located on another identical structure due to the same earthquake, will result in yet another graph. And if another earthquake occurs, then another set of different graphs will be obtained. The engineer must therefore consider the loading as inherently random. To get design information, the engineer considers a suitably large collection of recordings, called an ensemble, and examines their statistical properties.

If for all the recordings, at time t, the statistical properties of the loading are the same as at any other time t, the phenomenon is called a stationary process. An ergodic process is a special case of a stationary process. For an ergodic process, not only are the statistical properties the same at each time t, but for each recording the statistical properties of the loading are the same. For random processes the loading is typically described using a spectrum which, as shown in Figure 2e, has frequency on the x-axis rather than time.

1.3 Dynamic Equilibrium

In every problem of structural dynamics, regardless of the complexity of the model, the principal fact is that the forces involved are in equilibrium at any time, t. This is called D'Almbert's Principle of Dynamic Equilibrium and can be demonstrated simply, as shown in the diagram below.

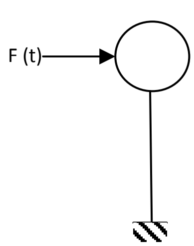

Figure 3a. Physical system

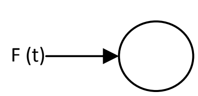

Figure 3b. Physical model or idealization

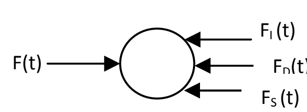

Figure 3c. Free body diagram (FBD)

Figure 3a shows a simple inverted pendulum under a time-varying force, F (t). It is basically a cantilever. Since this is a dynamic situation, inertia forces are generated that act on the mass. Figure 3b shows a model or idealization of the distribution of the mass – it is idealized as concentrated or "lumped" in the ball. That is, the mass of the stick portion of the pendulum is included in the lumped mass, and the model consists of just this mass. Figure 3c shows all the forces in the model. Clearly, at any time t,

$$F_I\,(t) + F_D\,(t) + F_S\,(t) = F\,(t) \tag{1.1}$$

F_I (t) is the inertia force. F_D (t) is the damping force. This represents the effect of internal friction, or the surrounding air mass, that causes the vibration to die away with time if the vibration is stopped. F_S (t) is the spring force. It represents the "springyness" of the system since the system is essentially a cantilever. The time-varying force F (t), is called the forcing function, or the excitation.

Equation 1.1 expresses the dynamic equilibrium of the system and is called the equation of motion, or the governing equation. By choosing a coordinate system, each of the forces can be expressed in terms of the displacement of the system and so equation 1.1 can be solved for this displacement, knowing which, all the forces on the left-hand-side (LHS) can be determined.

1.4 Modeling and Analysis Fundamentals

The previous section describes in a nutshell, what is involved in solving dynamics problems. However, in this example only one set of possible approaches was used to setup the model and its equations for solution, a process called formulation.

In general, the engineer has a specific set of decisions to be made that can affect both the accuracy and economy (i.e. time and resources required) of the entire solution process. These are briefly described in the following sections, as well as the emphasis that will be highlighted in this text. It should be understood however, that in many instances, especially with the ready availability of powerful computers and software, the final approach used may be just a matter of personal preference.

1.4.1 Lumped mass vs Distributed Mass

In the simple inverted pendulum example discussed previously, the mass of the system was idealized as a single lumped mass. This is because, since the inertia force is a central characteristic of dynamic analysis, and inertia is a property of mass, it is important to represent the mass and mass properties with sufficient accuracy. In contrast to a lumped mass idealization, the engineer can use a distributed mass idealization. This is closer to the physical reality of the system. However, the resulting mathematical model is considerably more complex compared to if a lumped mass approach is used. The accuracy relative to complexity is sufficiently high if a lumped mass idealization is used, that for many problems of practical interest to the civil engineer, this approach is sufficient. It is the principal approach utilized in this text, though a distributed mass approach is used for analyzing a beam in Chapter 4.

Another important idealization should be mentioned here, which is also made when the mass idealization is made. This is called the kinematic idealization. Recall the inverted pendulum example once more. It was mentioned that a coordinate system must be employed so that we can define the displacement of the mass. The displacement is called a degree-of-freedom of the mass, so our model is called a single degree-of-freedom (i.e. SDOF) model. It is possible for a mass to have more than one DOF. For example, the floor of a building can be idealized as a single mass, with 3-DOF: a displacement in the x-direction, a displacement in the y-direction, and the rotation of the floor in plan. Such a model is called an MDOF or multiple degree-of-freedom model. Another type of MDOF model is for example, a 2-dimensional building frame where each floor is idealized as a lumped mass, with its own single displacement. The bulk of this text is about SDOF models and is presented in Chapter 2. MDOF models are discussed in Chapter 3. It is interesting to note that for many problems of practical interest, an MDOF problem can be solved by using SDOF solutions.

1.4.2 Force vs Energy

For our inverted pendulum example, the governing equation is given by equation 1.1. This equation is in terms of force and, as will be shown in subsequent chapters, is a differential equation. This equation is to be solved for the dependant variable, the displacement. For relatively simple models, the solution of the differential equation is quite straightforward, but for more complicated models, it becomes much more difficult. In engineering analysis in general, it is usually much easier to solve governing equations if they are expressed using integrals rather than derivatives. The integral equations are said to be "weaker". The force equations can be converted to energy equations (using for example the Principle of Virtual Work) in which case the differential equations now become integral equations. It is more common however, for force, and hence differential, equations to be used for the types of dynamics problems of typical interest to civil engineers, so this form of the governing equations is used throughout this text.

1.4.3 Time vs Frequency

For our inverted pendulum example, the governing equation and its solution are functions of time and are said to be in the "time domain". However, for more complicated dynamic loading such as shown in Figures 2b and 2c, there is important information or patterns that are not obvious when expressed in the time domain. Rather than in the time domain, alternatively, the loading and response (i.e. the solution of the differential equation), can be expressed as functions of frequency. In this case, it is said that "frequency domain" analysis is being performed. This is made possible due to the branch of mathematics called Fourier Analysis. Solution in the frequency domain enables the engineer to determine the dominant frequencies in the loading and response and this has important application in the design of the system, the understanding of the behavior, or the classification of loading. Furthermore, it becomes relatively easy to readily determine the DAF of the response for a range of problems of practical interest.

With the exception of finding underlying patterns however, modern computer technology (via numerical analysis which is discussed in the next section) enables time domain analysis to provide all the information as a frequency domain analysis but in a single formulation. Frequency domain analysis also cannot be readily used for nonlinear problems. In this text, the emphasis is on time domain analysis though frequency domain analysis is presented for several types of dynamic loading, in Chapter 2.

1.4.4 Analytical vs Numerical Analysis

The governing equations of a dynamics problem can be solved analytically (i.e. as equations), also known as "in closed-form", but this is only possible for relatively simple dynamic loadings. For more complicated loading, numerical analysis must be used and the ready availability of powerful modern computers and software makes this quite straightforward. Numerical analysis algorithms are based on replacing a continuous variable by a series of small discrete steps and in this manner differential or integral equations are converted to algebraic equations, hence the necessity of computer technology. These procedures are based on series expansions (i.e. Maclaurin or Taylor series) and several have been developed for the direct or step-by-step integration of the governing differential equations of structural dynamics. Some of these are discussed in Chapter 2.

For MDOF analysis by one of the most popular approaches called mode superposition, it is first necessary to determine the vibration mode shapes of the structure and in engineering mathematics this is recognized as an eigenvalue problem. Numerical analysis algorithms have been developed for this as well, and is discussed in Chapter 3.

1.4.5 Linear vs Nonlinear

In structural dynamics the issue of linearity or nonlinearity is with respect to the third term in the LHS of equation 1.1 – the F_S (t) term. Recall that this is the spring force term. For many practical ranges of the performance of a structure, the stress can be considered directly proportional to the strain at any point in the structure. Hence for an SDOF system, $F_S(t) = K\,x(t)$, where K, the proportionality constant, is the spring constant or stiffness of the structure; x(t) is the time-varying displacement of the structure. However, there are important problems for which this assumption cannot be made. An example is the earthquake resistant design of buildings in which case, the engineer deliberately allows the structure to yield at a number of points within the structure. At those points the stress-strain relationship is nonlinear hence for the whole structure as well. Therefore in such a case, K is no longer a constant and depends on x(t). Hence $F_S(t) = K(x(t))\,x(t)$ which is clearly a nonlinear term.

The effect of nonlinearity is that another equation or procedure is required to evaluate K(x(t)) simultaneously with the other terms in the equation of motion and this makes the solution impossible by analytical procedures, hence frequency domain methods cannot be applied. When the problem is nonlinear, the typical approach is to use time domain modeling along with numerical analysis, and this is examined in Chapter 3.

It should be mentioned that because of the difficulties associated with a nonlinear dynamics problem, ingenious methods have been devised to convert the problem to an equivalent linear problem. One approach is to replace the F_S (t) term by changing the damping term F_D (t) in linear problem but in such a manner that the overall effect is the same as for the nonlinear problem.

1.4.6 Deterministic vs Random

In section 1.2 the specification of dynamic loading as a spectrum was discussed. Recall that this approach is required when the dynamics phenomenon is inherently random. In such a case, and only for linear problems, the governing equations have as the dependant variable, the moments of the response (e.g. the mean and standard variation).

From a civil engineering design perspective, this information can be used to determine a design load or response but the number used will be associated with a selected confidence level. For example, the load with a 2% probability of being exceeded. This is discussed further in Chapter 2.

Frequency domain solutions are extensively used for random dynamics problems, primarily because the governing equations are integral equations. This field is particularly interesting for researchers and is called Spectral Analysis.

1.5 Vibration Terminology

Certain terms are used in structural dynamics, in addition to those already presented, that are part of describing the various aspects of the topic. Some of the more fundamental terms are described as follows.

Period

Consider the vertical cantilever shown below in Figure 4a, and which is vibrating in a simple to-and-fro motion. The displacement at the tip, $x(t)$, is horizontal and has a maximum value of A when it bends to the far right, but –A to the far left. The horizontal displacement at any time t, is the projection onto a horizontal axis of a radius A of a circle, as shown in Figure 4b, as the radius rotates. If at time t=0, the radius is in the horizontal position, but thereafter, the radius rotates, say, anti-clockwise, then the time taken for the radius to make one complete revolution is the period. The typical symbol for the period is T and the unit is seconds (sec).

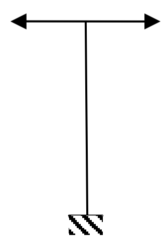

Figure 4a Vibrating cantilever

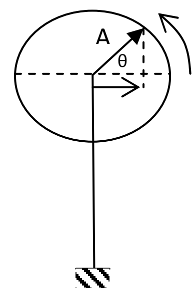

Figure 4b Vibration in terms of a rotating radius

Frequency

Referring to Figure 4b, the circular frequency is the angle from the horizontal that the radius rotates per second. The typical symbol for circular frequency is ω (or Ω) and its unit is radians per second or rps.

The rectilinear frequency is the number of complete revolutions made per second and is sometimes preferred, rather than the circular frequency, for describing the vibration. When the radius makes a complete circle it is said to have completed a cycle. The symbol for rectilinear frequency is f, and its unit is cycles per second or cps or Hertz (Hz). f is simply the inverse of T, so $f = 1/T$.

Since the radius rotates θ radians in time t, $\omega = \theta/t$, hence $\theta = \omega t$. The radius makes one complete circle in T seconds, therefore $2\pi = \omega T$. So,

$$\omega = 2\pi/T \qquad (1.2)$$

This is a fundamental equation of dynamics.

One must be conscious about which frequency is being referred to in dynamics equations since many texts simple use the word “frequency” for either the circular or rectilinear frequency.

Harmonic

The simple to-and-fro motion of the cantilever in Figure 4a is called harmonic motion. In terms of the rotating radius, as described previously, the displacement of the cantilever is the horizontal projection of the radius so is given by,

$$x(t) = A \cos \theta = A \cos \omega t \qquad (1.3)$$

The vibration of simple continua such as strings can be shown to be composed of the summation of equations such as 1.3 but each with a different “A” value and “ω” value. When these components are arranged in increasing order of frequency, they are called the first harmonic, the second harmonic, etc.

Phase
The cosine and sine functions have exactly the same shape, but they have different values for the same angle, ωt. Therefore, the cosine and sine functions will be identical if one is shifted on the angle-axis relative to the other. This shift or difference in angle is called the "phase difference", or just the "phase" for short. Hence the observation from trigonometry that $\cos \theta = \sin (\theta + \pi/2)$, where $\pi/2$ is the phase difference between the cosine and sine functions.

The phase angle is very useful in dynamics for representing loading or response equations in the most compact form.

Time Series
The shown in Figure 2, loading is frequently expressed as a function of time. This is also the case for system responses. The entire set of data is sometimes referred to as a time series or a record.

Magnitude
For the case of harmonic motion shown in Figure 4, the magnitude of the displacement is ±A. The magnitude of a time series is its peak or maximum value and is particularly important to civil design engineers since a structural system must be designed for the peak load or displacement that the system will experience during its service life.

For systems under the types of loading shown in Figures b, c, and d, both the loading and responses are typically mathematically represented in the frequency domain using summations of harmonics expressed in terms of a complex number for each frequency. In this case, the magnitude is the absolute value or modulus of the complex number.

Average
In several cases, the average or mean value of a time series, or the average of a set of values recorded at different points on a system, but at the same time t, is useful. For example, in the study of random vibrations, the "mean plus x-sigma" represents a certain probability of being exceeded, where x is typically between 1 and 3. Also, when representing a time series as summations of harmonics, the average value must also be added to complete the representation.

Root Mean Square
The root mean square or rms value of the displacement associated with a vibration is a measure of the energy of the vibration. The mean square displacement is the average of the squared values of the time series, or over a time interval. The rms is the square root of this value, and is measured in rms meters.

Bandwidth
A bandwidth is a range of frequencies that may be of particular significance in the dynamic loading or response of a system. For example, a particular bandwidth may be responsible for causing the highest DAF. For dynamic loading, its spectral representation is called "wide-band" if the ordinate (e.g. rms value) is the highest and fairly constant over a wide range of the frequencies relevant to describing the loading, as shown in Figure 2e. Earthquake loading is wide-band, whereas loading from a vibrating machine is typically narrow-band.

Octave
An octave is a frequency band in which the upper limit is twice the lower limit. So if frequencies are divided into bands of 10-20 Hz, 20-40 Hz, 40-80 Hz, then each of these bands is an octave.

Filter
This term is from the field of Signal Processing which shares many of the governing equations of linear structural dynamics. In Figure 1, the middle box is a filter – it converts the input vibration to another type of vibration as output. A high-pass filter reduces the values in the lower bands to very small values, and a low-pass filter does the opposite.

Practical work in structural dynamics involves taking measurements as time series from different points on a structure. However, because of computer technology the equipment uses digital electronics and the time series is the results of sampling at a certain time interval. Therefore, the terminology of Digital Signal Processing (DSP) is widely used in this aspect of structural dynamics and because of the sampling, certain phenomena may arise that the civil engineer must be aware of for proper use of the equipment, and interpretation of the resulting data.

1.6 The Basic Toolbox

The basic toolbox is comprised of the most fundamental methods or functions the engineer applying structural dynamics is likely to use. These are presented in the following table.

METHOD/ FUNCTION	**FUNCTION NAME**	**NOTES**
Fourier Series (M)	Not Applicable (NA)	Used to represent a periodic load, and the response due to a periodic load, as the sum of a series at discrete frequencies. Characterized by A, B coefficients, which are the basic structure of the Fourier Series.
Fourier Integral (M)	NA	Used to obtain frequency domain response due to a non-periodic load. The forcing function is part of the integral and the forcing function can be expressed as a time series of numerical values, or in closed-form. Represents the amplitudes of the exponential form of the Fourier Series.
Symbolic Integration (F, M)	**Int**	Can be used to get functions for A, B of the Fourier Series for closed-form solutions. The input is a symbolic expression and it returns the integral as a symbolic expression.
Absolute value (F)	**Abs**	Gives the absolute value of a complex number: a + ib, which can be used for a compact form of the Fourier Series. The input is the pair of numerical a, b numerical values and it returns a numerical value.
Phase (F)	**Angle**	Gives the phase (lag) angle between a, b. Returns a numerical value. The input is the pair of numerical a, b values of a complex number and it returns a numerical value.
Transfer function or Force Response Function (FRF) (F)	H	The DAF = |H| = **Abs(**H). If the input time series is expressed with t as a continuous function, then H represents an analogue filter. But if t is a set of discrete time values, then H represents a digital filter. In the former case, the MATLAB function that

		returns H is **freqs( ),** and for the latter is **freqz ( )**. In both cases, the input to H is a symbolic fraction, and they return numerical values.
Direct Fourier Transform (M)	P (sometimes F)	This is the closed-form frequency density of a time series and is used to calculate its amplitude spectrum, especially for hand-calculations, which is possible for simple dynamic loading. Cannot be directly calculated by a single MATLAB function, but a script can be developed, based on the **Int ( )** function, to get P (A, B also) in closed-form.
Fast Fourier Transform (F)	**FFT**	Gives the magnitude vs frequency of a given time series where the time is as a set of discrete equally spaced values. This is the numerical version of function P and is necessary for more complicated loading. The input is a set of time series numerical values, and it returns the numerical values of the amplitude spectrum.
Runge-Kutta 4^{th} Order Numerical Integration (M, F)	**Ode45**	Implements the Runge-Kutta 4^{th} or 5^{th} Order procedures of numerical integration of a set of 1^{st} order ODEs. The input is the symbolic expression of the DEs, the interval, and the y values at the left end of the interval. It returns the numerical values for y over the interval. Can be used to solve the ODEs representing the equation of motion.
Eigenvalue/Eigenvector Analysis (M, F)	**Eig**	For linear MDOF dynamics, the modal superposition method is widely used. An eigenvalue/eigenvector analysis gives the mode shape and its frequency. The input is the mass and stiffness values associated with each DOF.
Graphical Plotting (M, F)	**Subplot** and **Plot**	Though not a dynamics method, computer graphics is indispensible for displaying the input loads and response of dynamic analysis.

Students sometimes confuse the terms “closed-form”, “numerical”, “discrete”, and “continuous”. A closed-form solution means that the values can be calculated using a symbolic expression. A numerical solution means that the solution can only be obtained as a set of number values. “Discrete” means that the y-axis values only exist at specific x-axis values. An example is the spectrum of a Fourier Series, which only has values at specific frequencies but is zero in-between. “Continuous” means that any value on the x-axis has a value on the y-axis, hence when plotted a smooth curve is obtained. An example is the spectrum of a Fourier Integral which has values at all frequencies. Note that it is possible to have a closed-form solution for discrete spectra. This means that at the frequency in question, the value is given by a symbolic expression. Furthermore, even though a symbolic expression may exist, it is implemented in a computer program by calculating the value at discrete values, usually via a loop.

Computer Software

MATLAB
The methods/functions presented above are usually implemented using computer software, especially for practical problems. A popular software framework for math calculation is MATLAB by The Mathworks Inc. The functions in **bold** type in the table above are MATLAB command names. MATLAB is mainly a set of functions grouped into sets by topic and each set is called a “toolbox”. MATLAB comes with a basic collection of toolboxes but many others are available as options. MATLAB is a very hi-level language designed around the matrix as the fundamental data type so input data should be vectorized. MATLAB is an interpreted language and the commands can be collected together as a “script” file with extension .m which is in effect a computer program. MATLAB is available at many universities but can be pricy, even for the student-edition.

GNU Octave
Octave is a MATLAB open-source (i.e. free) clone that has evolved to almost 100 % compatibility. It uses the same commands as MATLAB. Octave for Windows is run from the command line. The install files for the Octave exes (called binaries), can be downloaded from:

http://octave.sourceforge.net/

An online manual for Octave is available from:

http://www.gnu.org/software/octave/doc/interpreter/

MAXIMA
Octave’s compatibility with MATLAB is, however, only with respect to numerical computations. MAXIMA is an open-source symbolic mathematics program that can be downloaded from:

http://sourceforge.net/projects/maxima/files/Maxima-Windows/5.22.1-Windows/maxima-5.22.1.exe/download

ENGLTHA
ENGLHA (Earthquake Nonlinear and General Linear Time History Analysis) is a computer program developed by the author for teaching structural dynamics. It gives the time domain solution for any input loading for a SDOF system, and caters for nonlinearity due to inelasticity, and including hysteretic degradation.

In this text, both MATLAB or Octave for numerical work, MAXIMA for symbolic work, and ENGLTHA are used to illustrate problem solving and demonstrations.

1.7 Problem Solving Strategy

The characteristics of a given dynamics problem indicate which approaches are possible for solution. The diagram on the following page is an overall summary of the main steps required for solving SDOF linear dynamics problems. For the frequency domain, the approach gives peak values only.

Though covering only the SDOF case, as will be seen in subsequent chapters, the most widely used method of MDOF dynamic analysis superimposes the results of SDOF analysis, making the SDOF analysis methods critical for the solutions of many types of problems, even for continuum structures.

PROBLEM SOLVING STRATEGY MAP FOR LINEAR SDOF STRUCTURAL DYNAMICS