CVNG 3001 STRUCTURAL ENGINEERING

SEISMIC LOADS – HORIZONTAL DISTRIBUTION AND TORSION

OUTLINE OF TOPICS

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Delivery Media: ☑ Oral ☑ Blackboard ☑ Handouts ☐ Slides/Transparencies ☑ Internet

Equipment: ☐ Slide Projector ☐ Transparency Projector ☐ Computer Projector

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Objective: To enable a rational basis for horizontal distribution of the floor inertia force, with inclusion of torsional effects, in determining the seismic loads on a building.

Scope/Limitations: Rigid diaphragms. Building inertia torsion only.

Primary Approach: Procedure-Based; Graphics-Based; Example-Based.

TOPICS

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SEISMIC LOADS II– HORIZONTAL DISTRIBUTION WITH TORSION

1.0 Calculation of Horizontal Distribution IncludingTorsion

The objective is the horizontal distribution of the inertia force to the frames or walls that support the floors. The following procedure assumes all floors are rigid and approximately the same, but can be readily modified to cater for floors of different mass and/or stiffness. Since the centre of mass of the typical floor (CM) and the centre of rigidity (CR) do not coincide, torsional forces are induced on the floors. The torsion is converted to an equivalent load on the frames, distributed horizontally as well, and added to the distributed inertia force. Hence, the effect of the torsion is to increase the force on the floor. This must be done for each of the 2 directions of the building.

1.1 Formulae

Total Lateral Loads Per Frame:

For a frame in the x-direction (frame in the y-direction similar),

Total Lateral Load, $V_{ix} = V_{ix}' + V_{ix}''$ (1)

Where V_{ix}' = Direct inertia force for the frame (that passes through CM),
V_{ix}'' = Effect of torsion (due to eccentricity between CM and CR).

$V_{ix}' = V_x$ x 2D frame or wall relative stiffness (2)
$= V_x$ x (2D frame or wall base shear/building base shear) (3)

$V_{ix}'' = (y_i V_{ix}' / I_{pv}) M_t$ (4)

Where I_{pv} = Polar Moment of Inertia = $\Sigma x_i^2 V_{iy}' + \Sigma y_i^2 V_{ix}'$ (5)

M_t = Torsion = $e_y \times V_x$, (6)

e_y = eccentricity in the y-direction
= (calculated eccentricity +
accidental eccentricity of 0.05 x building dimension in y-dir) (7)
V_x = building inertia force (base shear),
x_i, y_i = co-ordinates of the frames measured from the CR as the origin.

To get e_y (and e_x) the location of the Centre of Rigidity (CR), (x_r', y_r') must first be determined. This is done by taking moments about a convenient origin (typically the lower left-hand corner of the floor).

Therefore, $x_r' = \Sigma x_i' V_{iy}' / V$. Similarly, $y_r' = \Sigma y_i' V_{ix}' / V$ (8)

1.2 Procedure

The procedure for the calculation of the horizontal distribution including torsion (for a building with approximately equal floors) is:

Given: The building's base shear and vertical distribution. For each direction, either: (a) the relative lateral stiffness of each frame or wall (from base to roof), or (b) the force on the individual frames or walls.

Step 1. Develop a rectangular co-ordinate system ($x^{'}$, $y^{'}$) with the origin at the lower left-hand corner of the floor. Then from eqn (8), determine the location of CR.

Step 2. Draw the plan of the floor and determine the centre of mass, CM. A convenient tool such as AUTOCAD 2000 can be used to do this, if there are significant openings in the floor (such as for elevator shafts or stair wells).

Step 3. Develop a rectangular co-ordinate system (x, y) with the origin at the CR. Then label the frames or walls in the x and y directions and determine the co-ordinates for the frames or walls (i.e. x_i, y_i,).

Step 4. Determine the eccentricity from eqn (7).

Step 5. Determine the torsion from eqn (6).

Step 6. Determine the polar moment of inertia from eqn (5).

Step 7. For the frames or walls in the x-direction, substitute in equations (4) to (1).

Step 8. For the frames or walls in the y-direction, re-do step 5, and sub in the equations parallel to (4) to (1).

Step 9. Distribute the forces to the floor of each frame or wall in proportion to the vertical distribution.

2.0 Relative Stiffness

2.1 Calculation of Relative Stiffness

2.1.1 <u>Formulae</u>

This approach should not be used for building of more than 7 stories.

<u>For frames with fixed-end columns:</u>

Individual column (from base to roof level) lateral stiffness, $k_c = 12EI/L^3$

Where E = young's modulus, I = cracked moment of inertia, L = column length.

Frame stiffness, k_{fi} = no of columns x k_c . (Similar if frame is of different size columns).

Relative lateral stiffness for frame i, kr_{fi} = frame stiffness, $k_{fi} / \Sigma k_{fi}$ for all frames

<u>For cantilever walls:</u>

Individual wall (from base to roof level) lateral stiffness, $k_w = 3EI/(h^{3[}(1+0.6(1+\nu)d^2/h^2])$

Where E = young's modulus, I = cracked moment of inertia, h = wall height
d = wall width, b = wall thickness, and ν is the wall poisson ratio.

Wall stiffness, k_{wi} = no of walls x k_w . (Similar if the elevation has different size sizes or types of walls).

Relative lateral stiffness for frame i, kr_{wi} = frame stiffness, $k_{wi} / \Sigma k_{wi}$ for all frames

2.1.2 <u>Linked-Frame Model</u>

This approach is more accurate than the formula method but requires structural analysis software.

In this approach, the frames (or walls) are all lined up but joined at each floor by rigid links to form one model. The vertically distributed inertia forces at each floor (from the base shear force analysis) are then applied at one end.

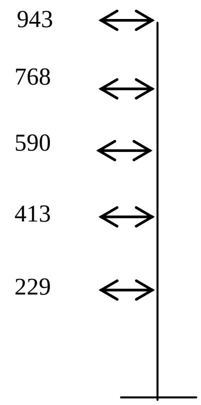
This model is analysed (by plane-frame analysis on a computer) to obtain the support reactions, at the base, for each frame (or wall).

The relative stiffness of the frame (or wall) in question is then the ratio of the horizontal reactions for that frame (or wall) divided by the total base shear.

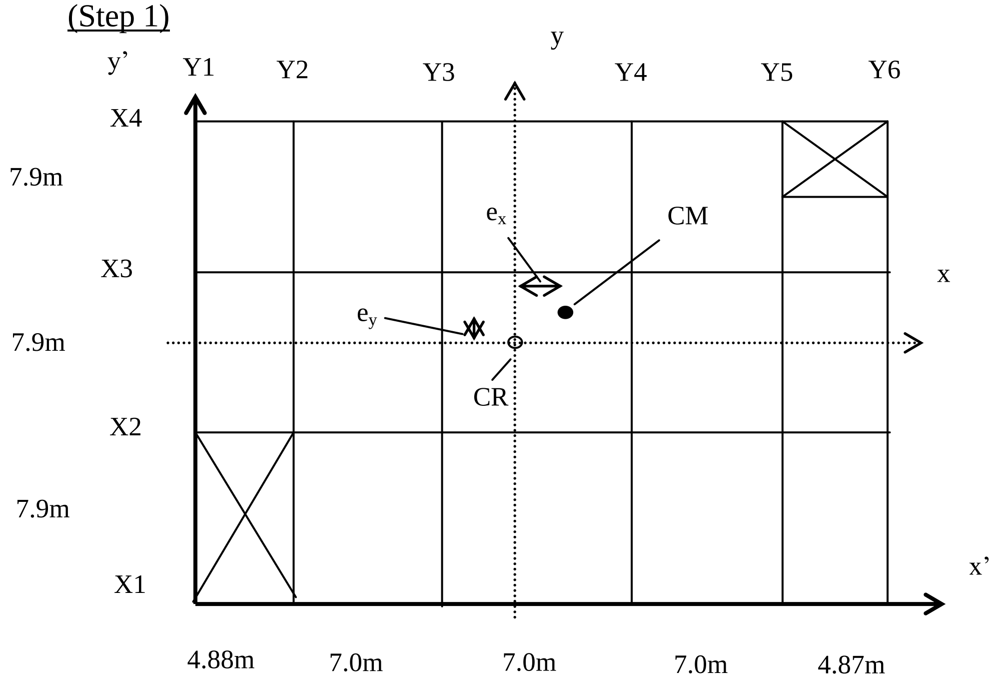
3.0 Example Calculation of Horizontal Distribution Including Torsion

A 5-storey framed building is comprised of similar floors 30.75m x 23.7 m. The base shear force analysis indicates a base shear of 2946 kN distributed vertically as shown. A linked-frame model was used to determine the distribution of the direct base shear to the frames as:

For frames X1 to X4, 1125.6 kN, 593 kN, 589 kN, 635 kN.
For frames Y1 to Y6, 602.5 kN, 506.9 kN, 521.6 kN, 520.9 kN, 395.9 kN, 395.1 kN.
Find the earthquake forces at the floors of the individual frames in the building's longer direction, including torsion effects.



<u>(Step 1)</u>



The $x_i^{'}$, (for the Y frames) are: 0,4.88, 11.88, 18.88, 25.88, 30.75 m
The $y_i^{'}$ (for the X frames) are: 0, 7.9, 15.8, 23.7m

From equ (8), the location of CR is calculated as:

$x_r^{'} = \Sigma x_i^{'} V_{iy}^{'} / V =$
$[(4.88x506.9)+(11.88x521.6)+(18.88x520.9)+(25.88x395.9)+(30.75x395.1)]/2946 =$
13.88 m

$y_r^{'} = \Sigma y_i^{'} V_{ix}^{'} / V = [(7.9x\ 593)+(15.8x589)+(23.7x635)]/2946 = 9.86m$

<u>(Step 2)</u>
The location of CM has been determined from AUTOCAD as being 16.13m horizontally from the bottom left corner, and 12.02m vertically.

<u>(Step 3)</u>
The x_i and y_i for the respective frames are therefore:

For the Y-frames:

$x_1 = -13.88m$ $x_2 = -9.0m$
$x_3 = -2.0m$ $x_4 = 5m$
$x_5 = 12m$ $x_6 = 16.87m$

For the X-frames:

$y_1 = -9.86m$ $y_2 = -1.86m$
$y_3 = 5.94m$ $y_4 = 13.84m$

<u>(Step 4)</u>

Eccentricities:

$e_x = 16.13 – 13.88 = 2.25$ m $e_y = 12.02 – 9.86 = 2.16m$

Accidental eccentricity in the x-direction = 0.05 x 30.75 = 1.54m
Accidental eccentricity in the y-direction = 0.05 x 23.7 = 1.18m

<u>(Step 5)</u>

$M_t = (2.16 + 1.18) \times 2946 = 9840$ kNm

<u>(Step 6)</u>

$I_{pv} = \Sigma x_i^2 V_{iy}^{'} + \Sigma y_i^2 V_{ix}^{'}$

$= (13.88^2 \times 602.5) + (9^2 \times 506.9) + (2^2 \times 521.6) + (5^2 \times 520.9) + (12^2 \times 395.9) + (16.87^2 \times 395.1) + (9.86^2 \times 1125.6) + (1.96^2 \times 593) + (5.94^2 \times 589) + (13.84^2 \times 635)$

$= 595{,}817.9$ kNm^2

<u>(Step 7)</u>

For frame X1:
$V_{1x}'' = (-9.86 \times 1125.6/595{,}817.9) \times 9840 = -183.3$ kN
Hence total lateral load = 1125.6 – 183.3 = 942.3 kN

For frame X2:
$V_{2x}'' = (-1.96 \times 593/595{,}817.9) \times 9840 = -19.2$ kN
Hence total lateral load = 593 – 19.2 = 573.8 kN

For frame X3:
$V_{3x}'' = (5.94 \times 589/595{,}817.9) \times 9840 = 57.8$ kN
Hence total lateral load = 589 + 57.8 = 646.8 kN

For frame X4:
$V_{4x}'' = (13.84 \times 635/595{,}817.9) \times 9840 = 145.1$ kN
Hence total lateral load = 635 + 145.1 = 780.1 kN

<u>(Step 9)</u>

For frame X1 only; the others are left as an exercise for the student.

From the vertical distribution, the inertia force per floor from the top down is 32%, 26.1%, 20%, 14%, and 7.9%. Hence applying these ratios to 942.3 kN, we get,