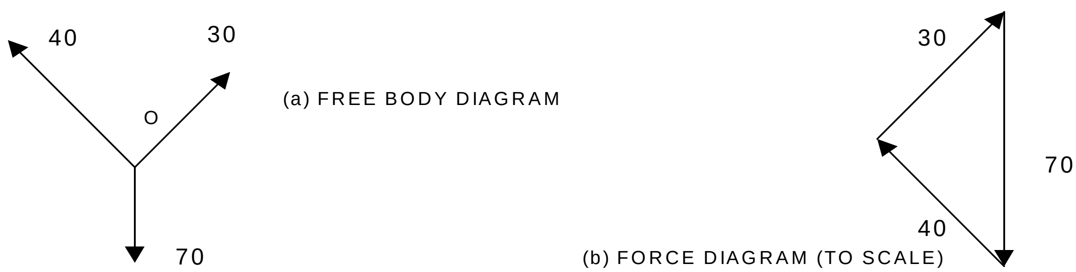
CVNG 1000 MECHANICS OF SOLIDS

SELECTED EXAMPLES

Provided by Prof. A. K. Sharma

FORCES AND STATIC EQUILIBRIUM

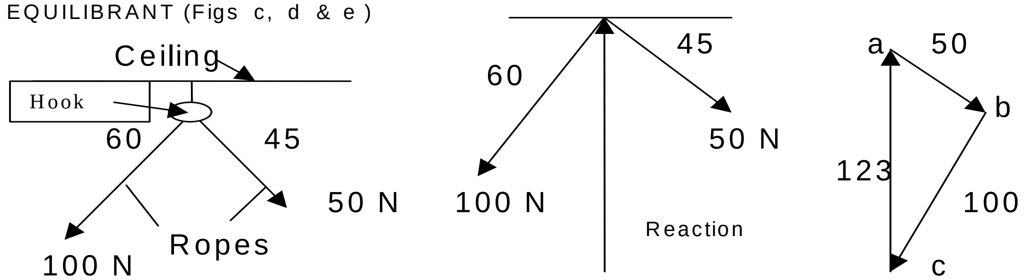
1. Triangle of Forces (Figs a & b)



(a) FREE BODY DIAGRAM

(b) FORCE DIAGRAM (TO SCALE)

2. EQUILIBRANT (Figs c, d & e)



(c)

(d) Free body diagram

(e) Force Diagram

"ca" - ceiling reaction in magnitude and direction which is 123 N; since reaction provides the force for equilibrium, it is known as EQUILIBRANT

The above examples illustrate three forces (two ropes and a ceiling reaction) which keep the hook in equilibrium

3. Resultant (Fig. c, d & e) In the above example a pull of 123 N equal and opposite to the ceiling reaction (Equilibrant) keeps the hook in equlibrium. This is known as resultant.

4. Parallelogram of Forces (Figs f, g)

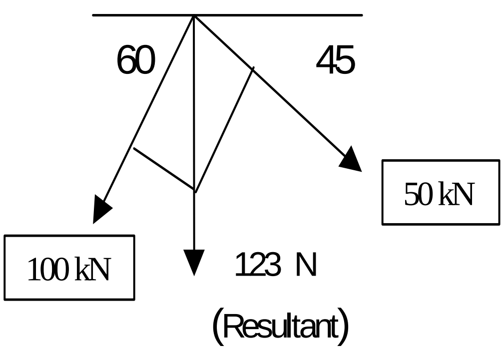


Fig. f

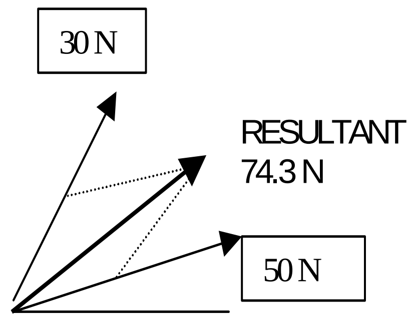
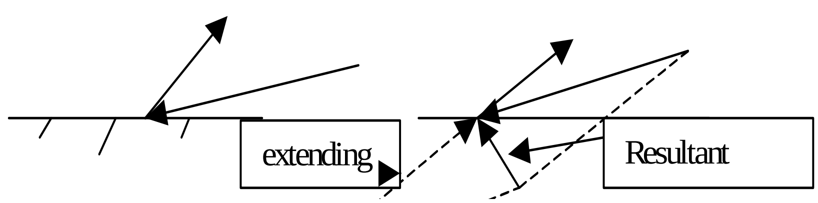


Fig. g

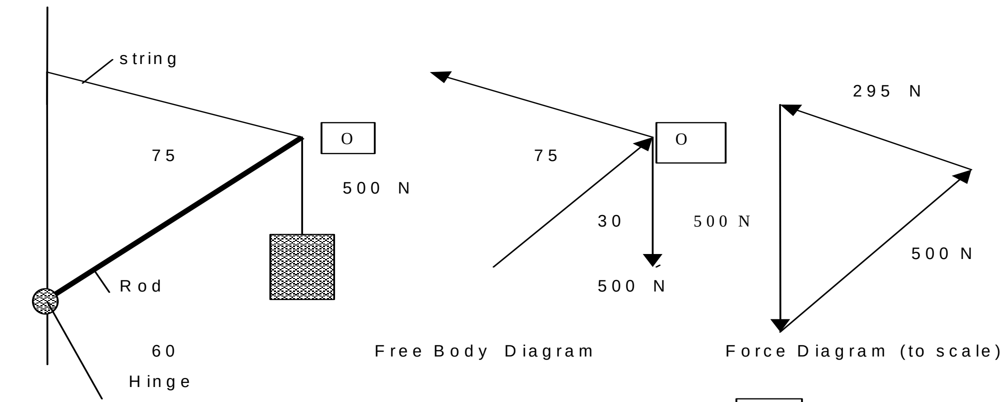
Draw a parallelogram of forces to scale and obtain the diagonal in in magnitude and direction as the resultant.

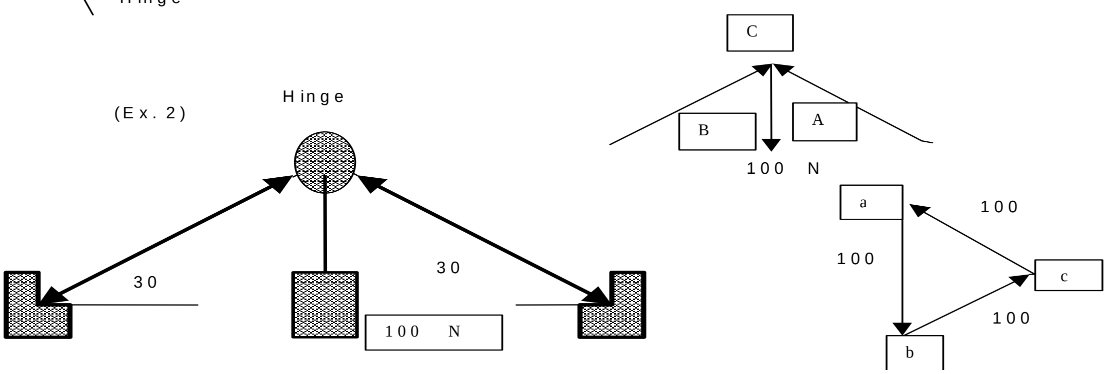
Fig. h



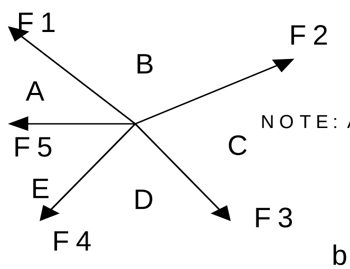
5. Hinged or Pinned Joints (Ex. 1)

To Find: FORCES in the Rod and String

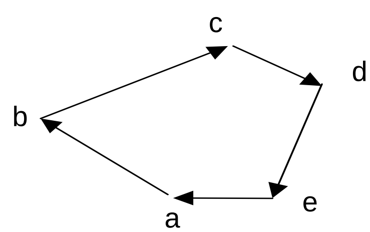




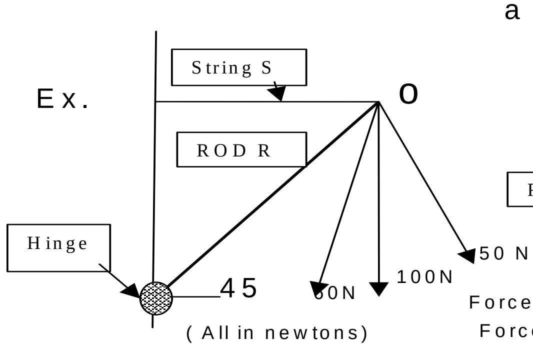
6. POLYGON OF FORCES : When number of forces meeting at a point are in equilibrium they are represented in magnitude and direction by a closed polygon . If not - the closing line represents the RESULTANT.



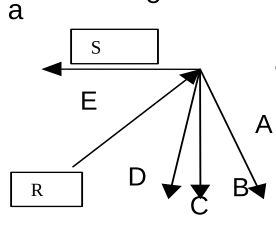
NOTE: AB, BC, CD etc are referred as BOW S NOTATION



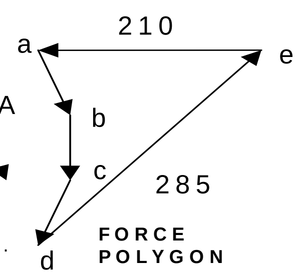
Ex.



(All in newtons)



Free Body Diag.



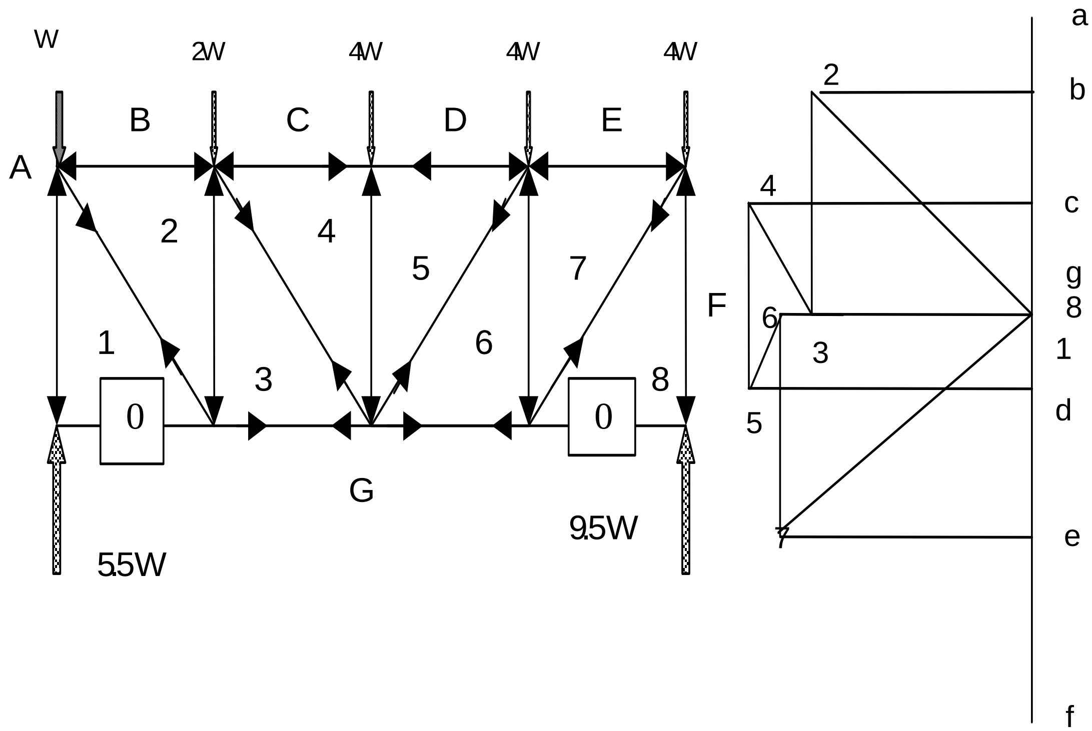
FORCE POLYGON

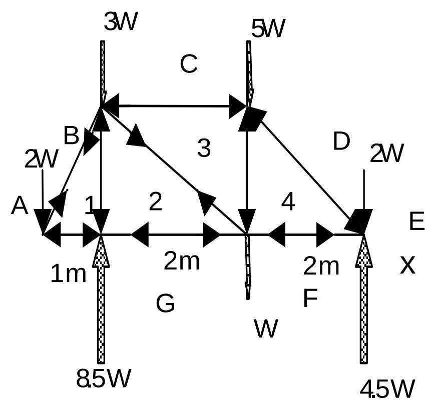
Force DE pushes towards O (Compressive)

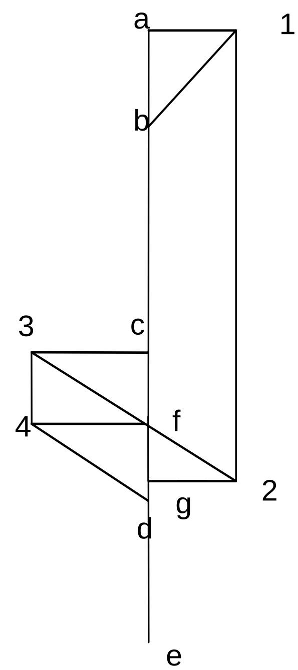
Force EA pulls away from O (Tensile)

STATICALLY DETERMINATE TRUSSES

EX2

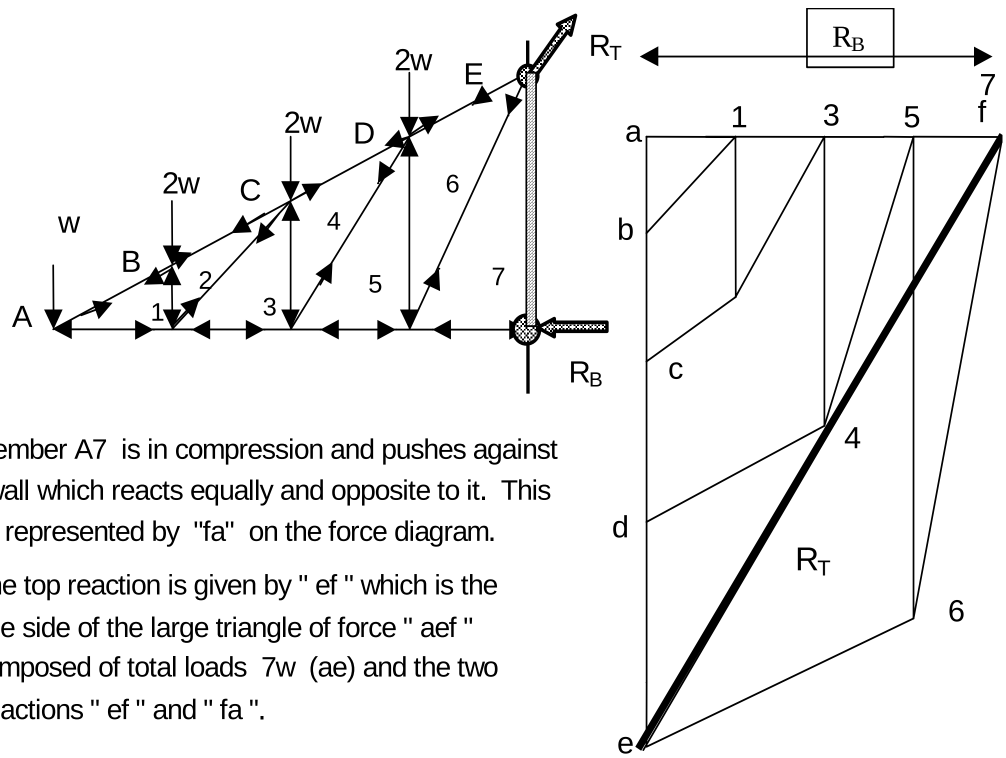






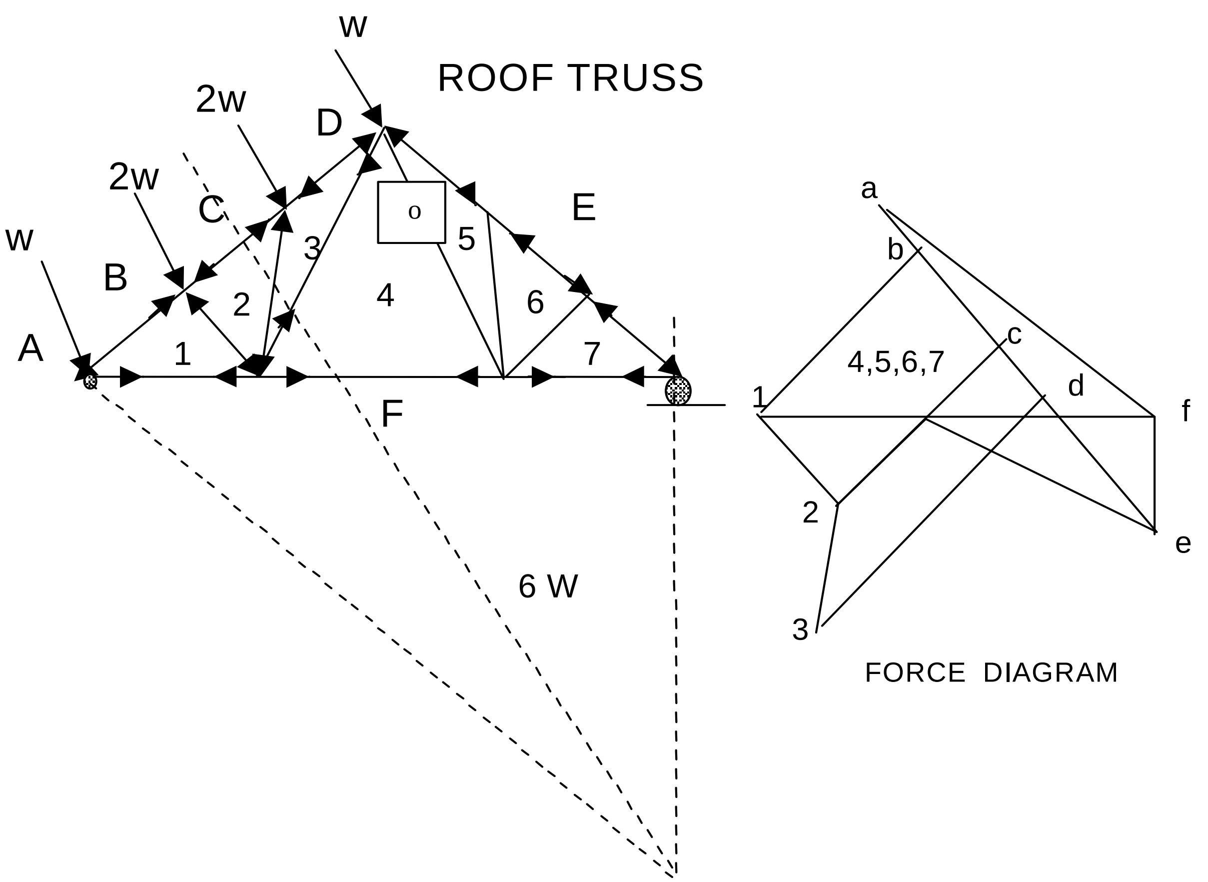
For reactions, Take moments about X

Reactions are as shown.

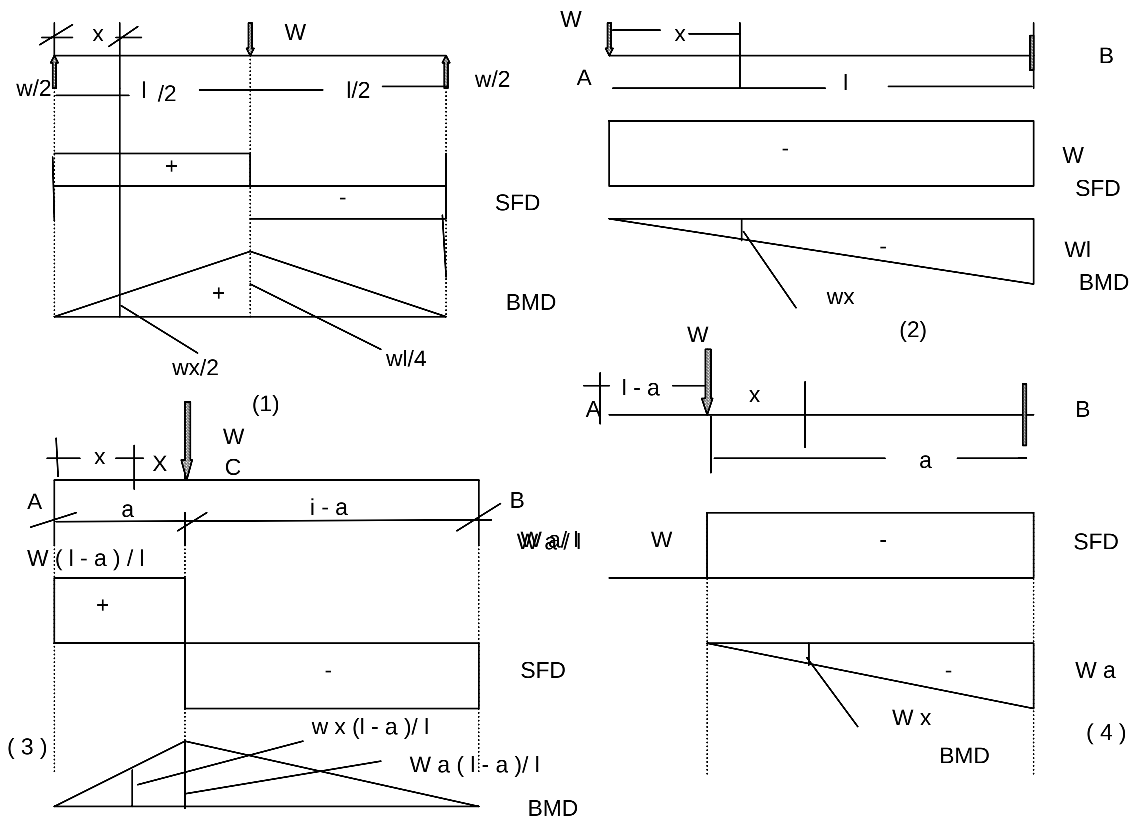


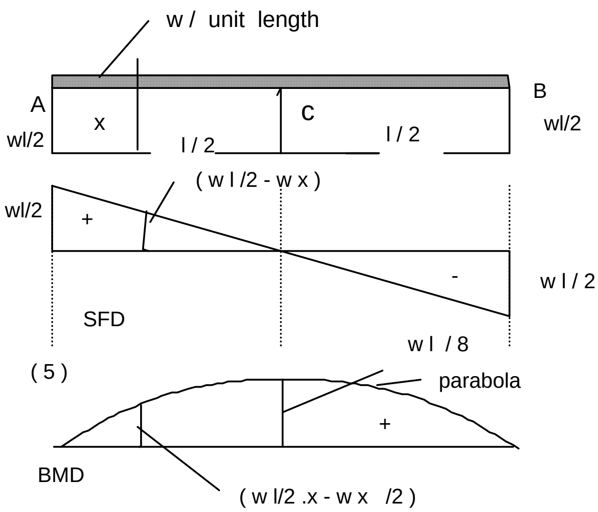
Member A7 is in compression and pushes against wall which reacts equally and opposite to it. This is represented by "fa" on the force diagram.

The top reaction is given by " ef " which is the one side of the large triangle of force " aef " composed of total loads 7w (ae) and the two reactions " ef " and " fa ".

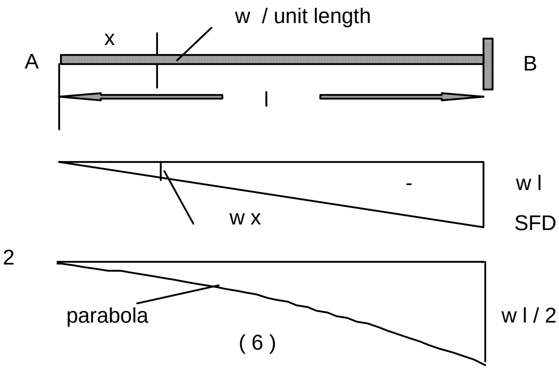

ROOF TRUSS
w
2w
2w
w
A
B
C
D
E
F
o
1
2
3
4
5
6
7
6 W
a
b
c
d
f
e
1
2
3
4,5,6,7
FORCE DIAGRAM

STATICALLY DETERMINATE BEAMS

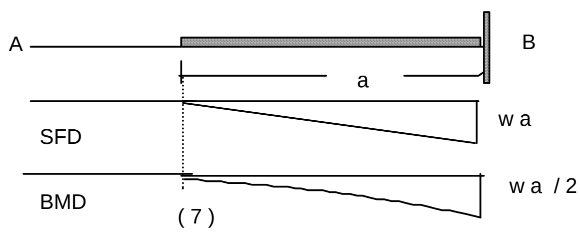



w / unit length
A
B
x
C
wl/2
l / 2
l / 2
(w l /2 - w x)
wl/2
+
-
w l / 2
SFD
w l / 8
parabola
+
BMD
(w l/2 .x - w x   /2)

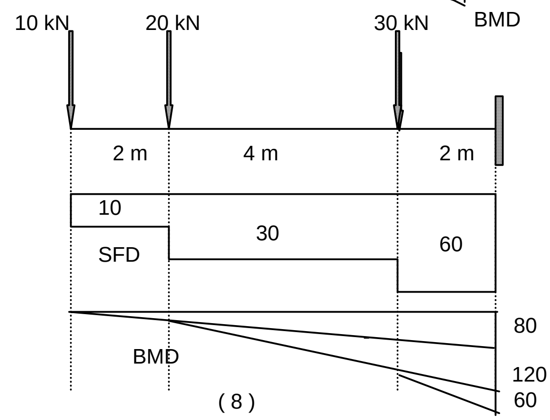
(5)


w / unit length
x
A
B
wl/2
l
-
w l
w x
SFD
parabola
w l / 2
BMD

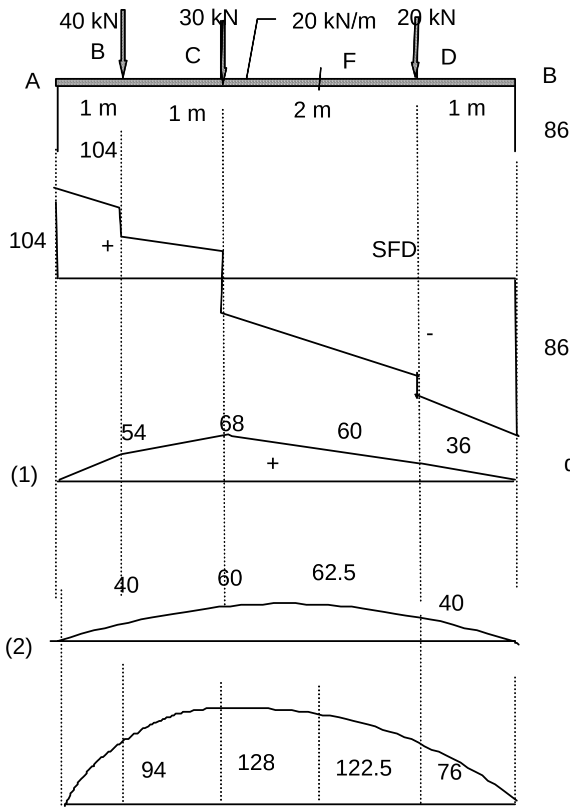
(6)


A
B
a
SFD
w a
BMD
w a / 2

(7)


10 kN
20 kN
30 kN
2 m
4 m
2 m
10
30
60
SFD
80
BMD
120
60

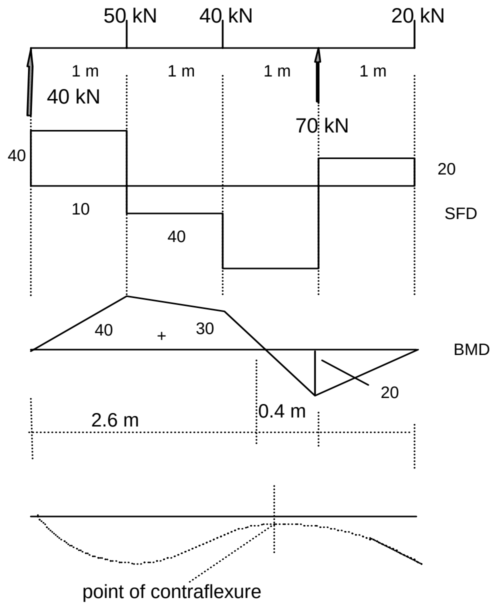
(8)



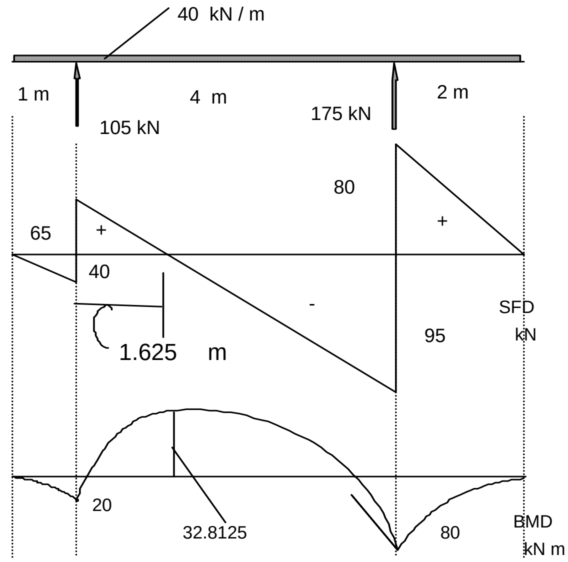
due to concentrated load

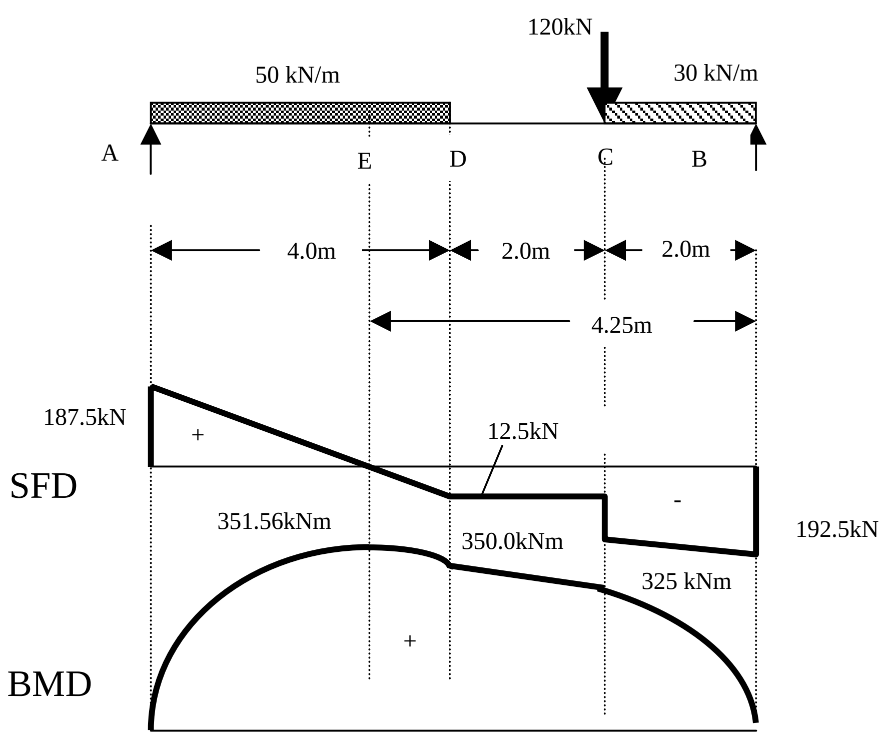
due to uniformly distributed load

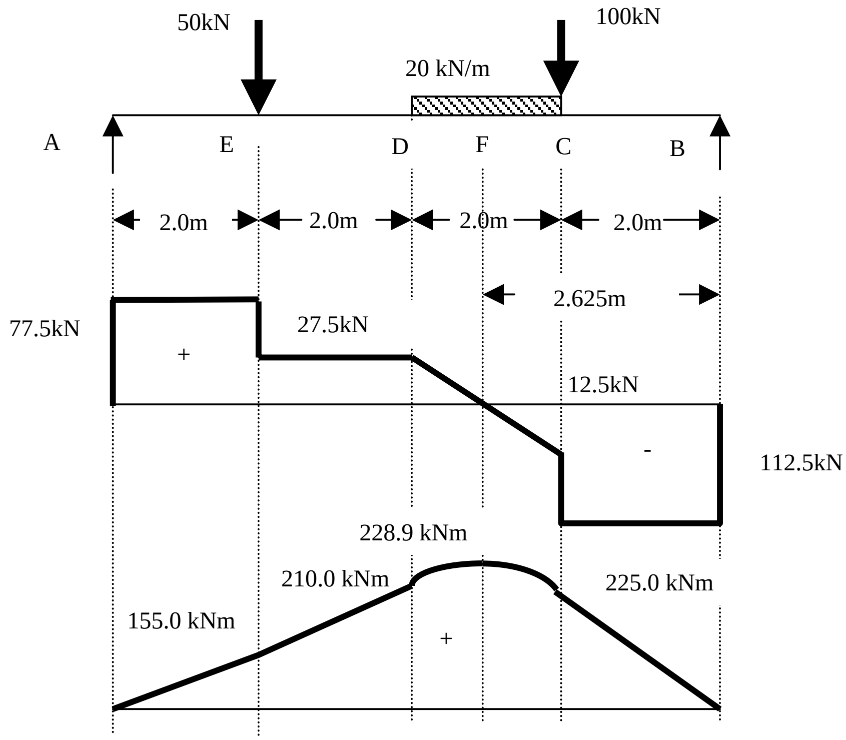
superimposing (1) & (2) BMD to get final BMD

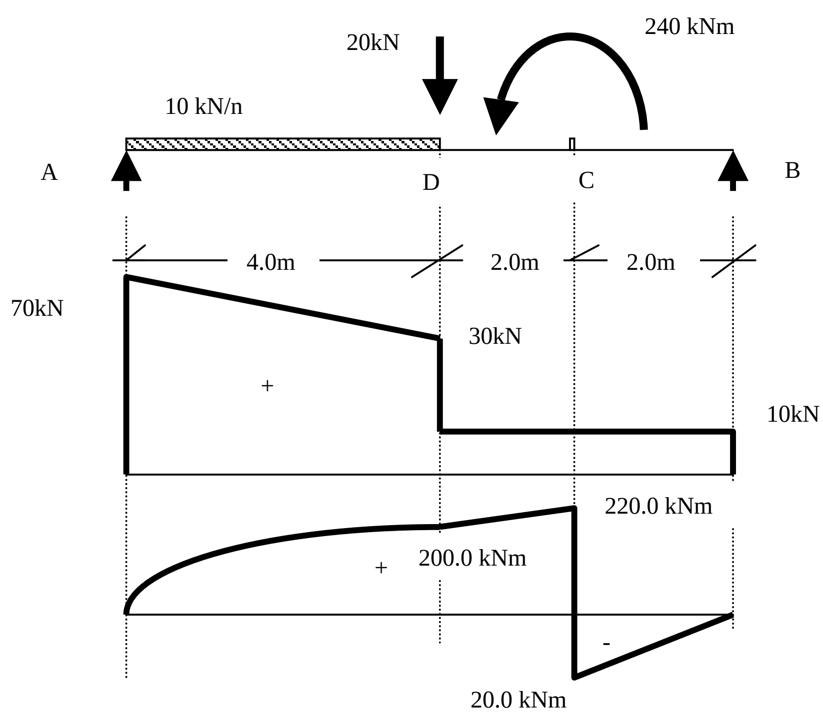


DEFLECTED FORM OF THE BEAM



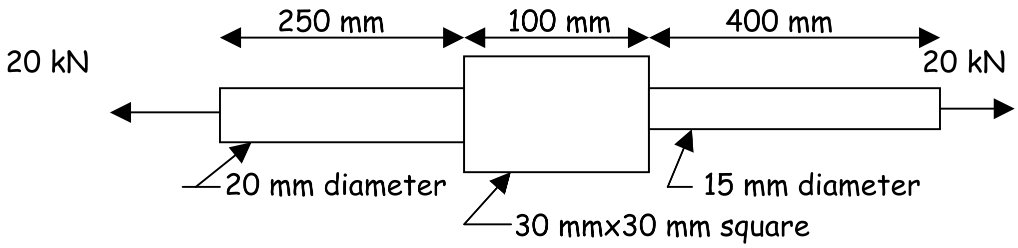



50kN
100kN
20 kN/m
A
E
D
F
C
B
2.0m
2.0m
2.0m
2.0m
2.625m
77.5kN
27.5kN
+
12.5kN
-
112.5kN
228.9 kNm
210.0 kNm
225.0 kNm
155.0 kNm
+


240 kNm
20kN
10 kN/n
A
D
C
B
4.0m
2.0m
2.0m
70kN
30kN
+
10kN
220.0 kNm
+
200.0 kNm
-
20.0 kNm

STRESS, STRAIN IN TIES AND STRUTS

Example 1.



Shaft shown above is loaded and has cross sectional dimensions as shown above. Calculate the total extension of the shaft if Young's Modulus is 210 GN/m^2.

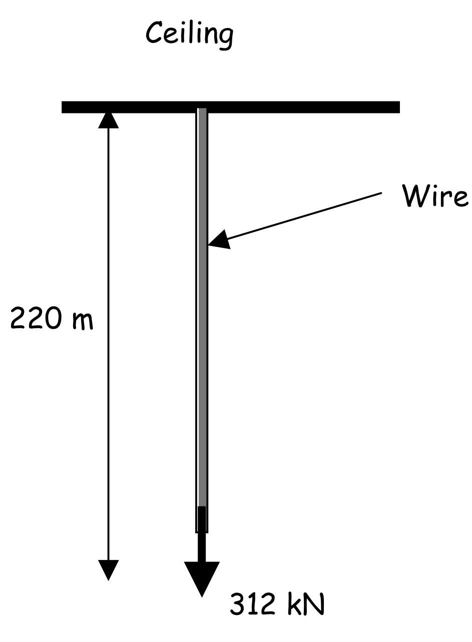
$$E = 210\ GN/m^2 = 210 \times 10^9\ N/m^2 = 210 \times 10^3\ N/mm^2$$

Area of 20 mm diam. Bar = $\pi/4 \times 20^2 = 314.159\ mm^2$
Area of square section = $30 \times 30 = 900\ mm^2$
Area of 15 mm diam. Bar = $\pi/4 \times 15^2 = 176.715\ mm^2$

Elongation = $P/E\ \{L_1 / A_1 + L_2 / A_2 + L_3 / A_3\}$

Total Extension = $(20\times10^3) / (210 \times 10^3)\ \{ (250/314.159) + (100/900) + (400/176.715) \}$ = 0.301945 mm

Example 2



Given: Permissible stress in the wire = 70 Mpa; density of wire = 7850 kg / m^3 and E = 210×10^3 N/mm^2. Calculate the minimum diameter of the wire and the extension of the wire.

We have the area of the wire = $F/\{\sigma_p - wL\}$

Density of wire, $w = 7850 \times 9.81 \times 10^{-9}$ N/mm^3

Hence Area $A = 312 \times 10^3 / \{ 70 - 7850 \times 9.81 \times 10^{-9} \times 220 \times 10^3 \} = 5880$ mm^2

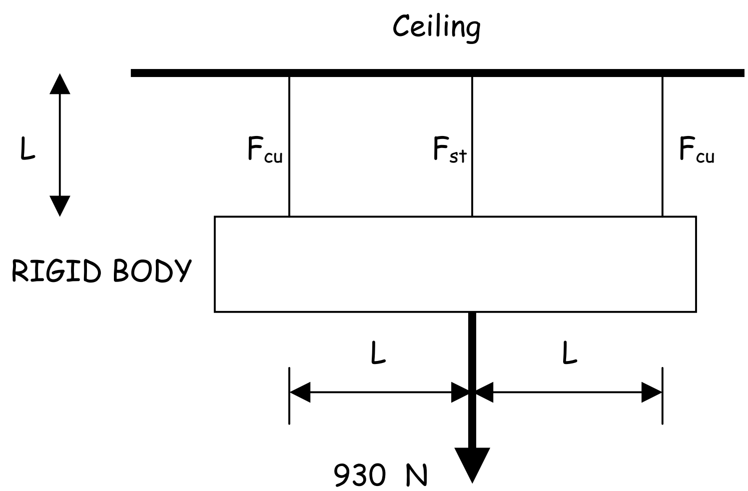
Thus $\pi /4 \times d^2 = 5880$ or $d = 86.525$ mm

Now extension = $FL/AE + WL/2AE = L/AE [F + wLA/2]$

Extension = $220 \times 10^3 / (5880 \times 210 \times 10^3)[312 \times 10^3 + (7850 \times 9.81 \times 10^{-9} \times 220 \times 10^3 \times 5880) /2$

= 64.46 mm

Example 3



Rigid Body weighs 930 N ; it is supported by three vertical wires (central being of steel and the outer wires are of copper) as arranged in the figure. If the modular ratio = $E_{cu} / E_{st} = 8/15$ calculate the forces shared by each wire and the elongation of the wires if the body remains horizontal.

For vertical eqilibrium: $930 = 2 F_{cu}+ F_{st} = 2 \sigma_{cu} A + \sigma_{st} A = A (2 \sigma_{cu} + \sigma_{st})$
Since Length is same for each wire and that elongation is same the strain in each wire must be the same

Thus $\sigma_{cu} / E_{cu} + \sigma_{st} / E_{st}$ or $\sigma_{cu} = \sigma_{st} \{E_{cu} /E_{st} \}$

$930 = A (2 \sigma_{st} [E_{cu} / E_{st}] + \sigma_{st}) = A (2 \times[8 /15] \times \sigma_{st} + \sigma_{st}) = A \sigma_{st} [31/15]$
$930 = F_{st} [31/15]$ or $F_{st} = 450$ N and $F_{cu} = 240$ N

Check: $930 = 2 \times 240 + 450$ OK

Example 4 :

Cross sectional area of each wire = 320 mm^2 and Coefficient of thermal expansion, for steel, α_{st} = 12 x 10^{-6} / degree centigrade and copper α_{cu} = 18 x 10^{-6} / degree centigrade and E_{st} = 210 x 10^3 N/mm^2 and E_{cu} = 112 x 10^3 N/mm^2. Find the thermal stresses in each bar.

Using $\Sigma (\delta L/L - \alpha T) E A = 0$ Thus

Copper   Steel

$\{(\delta L/L - 18 \times 10^{-6} \times 100) \times 112 \times 10^3 \times 320\} + 2 \{ (\delta L/L - 12 \times 10^{-6} \times 100) \times 210 \times 10^3 \times 320 \} = 0$

$(\delta L/L \times 3.548 \times 10^7 - 64512) + (\delta L/L \times 13.44 \times 10^7 - 161280) = 0$

$\delta L/L \times 17.024 \times 10^7 - 225792 = 0$

$\delta L/L = 1326.3157 \times 10^{-6}$

Therefore stress in copper bar, $\sigma_{cu} = E_{cu} (\delta L/L - (\alpha T)_{cu}$
$= [1326.3157 \times 10^{-6} - 18 \times 10^{-6} \times 100] \times 112 \times 10^3 = -53.053$ N/mm^2 (compressive)

Stress in steel bar = σ_{st} = $[1326.3157 \times 10^{-6} - 12 \times 10^{-6} \times 100] \times 210 \times 10^3$
= + 26.526 N/mm^2 (tensile)

Check: Force in each steel bar = 26.526 x 320 = 8488 N
Force in copper bar = -53.053 x 320 = -16976.96 N

Σ Force = 0 OK

Example 5: A water main 800 mm in diameter contains water at a pressure head of 100 m. If the weight of water is 10 kN/m^3, find the thickness of the metal required for the water main. Take allowable stress of the water main metal as 20 Mpa.

Pressure of water inside the water main = p = wH $10 \times 10^3 \times 100 = 10^6$ N/m^2

Permissible stress is equal to (hoop stress as it is critical) = pr/t

Permissible stress = 20×10^6 N/m^2

Thus $t = pr/(20\times10^6) = (10^6 \times 0.4) / (20 \times 10^6) = 0.02$ m = 20 mm

Example 6: A boiler is subjected to an internal steam pressure of 2 Mpa. The thickness of the boiler plate is 20 mm and permissible stress is 120 Mpa. Calculate the maximum diameter, when the efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

Considering efficiency of joints we have
σ_H should not exceed 120 x 90/100 = 108 N/mm^2
σ_L should not exceed 120 x 40/100 = 48 N/mm^2

For p = 2 Mpa
Hoop stress σ_H = pr/t = 2 x r/20 = 108 or r =1080 mm

Longitudinal stress, σ_L = pr/2t = 2 r/(2 x 20) = 48 or r = 960 mm

Hence maximum allowable diameter = 2 x 960 = 1920 mm (Maximum diameter of the boiler is equal to the maximum value of the diameter given by the two relations above)

Example 7: A boiler shell is made of 15 mm thick plate having limiting tensile stress of 120 Mpa. If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively, determine

(a) The maximum permissible diameter of the shell for an internal pressure of 2 Mpa and
(b) Permissible intensity of internal pressure when the shell diameter is 1.5 m.
(c) Taking the limiting tensile stress = circumferentail (hoop) stress

σ_H = 120 Mpa; Now $\sigma_H = pr/(t \times \eta_H)$ or $120 \times 10^6 = 2 \times 10^6 \times r / (0.015 \times 0.7)$

r = 0.63 m or Diameter = 1.26 m

Now taking limiting tensile stress = Longitudinal stress, σ_L = 120 Mpa

Now $\sigma_L = pr/(2\, t\, \eta_L)$ or $120 \times 10^6 = 2 \times 10^6 \times r / (2 \times 0.015 \times 0.3)$

r = 0.54 m or Diameter = 1.08 m
Thus the maximum diameter of the shell is the minimum value of the above two cases. Therefore D = 1.08 m

(c) Taking limiting tensile stress = hoop stress (σ_H) = 120 Mpa

Using the relation $\sigma_H = p\, r /(t\, \eta_H)$ or $120 \times 10^6 = p \times 10.^6 \times 1.5 / (2 \times 0.015 \times 0.7)$

P = 1.68 Mpa

Taking limiting tensile stress = Longitudinal stress (σ_L)

Using the relation: $\sigma_L = p\, r / (2 \times t \times \eta_L)$ or $120 \times 10^6 = p \times 10^6 \times 1.5/(2 \times 0.015 \times 0.3)$

P = 1.44 Mpa

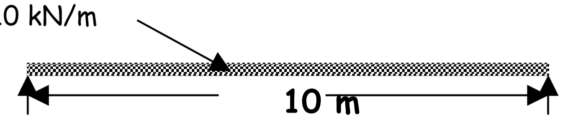
Hence in order both the conditions may be satisfied the maximum permissible internal pressure is equal to the minimum of the above two values.

Hence permissible internal pressure = 1.44 N/mm^2

STRESS/, STRAIN IN HOMOGENEOUS BEAMS

Example 1: A rectangular beam 10 m long is simply supported at its ends and is acted upon by a 10kN/m uniformly distributed load throughout its length as shown in the figure. Find the size of a beam having its depth is twice that of its width (d = 2b). Take the allowable bending stress of the material as 40 N/mm^2.

10 kN/m

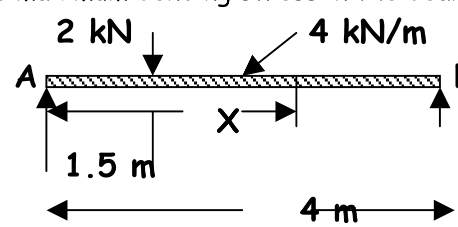


Solution: Maximum bending moment , $M_{max} = wl^2/8 = 10 \times 10^2 / 8 = 125$ kNm

We know $M_{max} = \sigma_{max} \times Z = \sigma_{max}\ I/y_{max} = \{ \sigma_{max} \times 1/12 \times b \times (2b)^3 \}/ (2b/2)$
$= \sigma_{max} \times 4/6 \times b^3$

Thus $125 \times 10^6 = \{ 40 \times 4/6\} \times b^3$ or $b^3 = \{125 \times 10^6 \times 6\}/160$
or b = 167.36 mm and d = 2 x b = 2 x 167.36 = 334.72 mm

Example 2: A 4 m long rectangular beam of size 100 x 200 mm is simply supported at its ends. If it is loaded with a UDL of 4 kN/m throughout its length and a concentrated load of 2 kN at a distance of 1.5 m from its left support, determine the maximum bending stress in the beam.



First calculate Reactions; $R_A = \{(4 \times 4 \times 2) + 2 \times 2.5\} / 4 = 9.25$ kN
$R_B = \{ (4 \times 4 \times 2) + 2 \times 1.5\} / 4 = 8.75$ kN
Check: $R_A + R_B = 4 \times 4 + 2 = 18$ kN ok

Let us consider a section at a distance X from A then

$M_x = 9.25 . X - 2 (X-1.5) - 4 . X . X^2 / 2 = 7.25 . X + 3 - 2.X^2$

M is maximum when $dM_X/dX = 0$ hence $7.25 - 4 X = 0$ giving X = 1.81 m

Thus Maximum Bending moment = $7.25 \times 1.81 + 3 - 2 \times (1.81)^2 = 9.5703$ kNm

I for the section $= 1/12 \{ 100 \times 200^3\} = 6.67 \times 10^7$ mm^4
thus section modulus, $Z = 6.67 \times 10^7 / (200/2) = 6.67 \times 10^5$

Maximum bending stress = M_{max} / Z = (9.5703 x 10^6)/ (6.67 x 10^5)
= 14.35 N/mm^2

Example 3: A floor has to carry a load of 5 kN/m^2. It is supported by wooden joists 150 mm x 300 mm size over a span of 5 mm as shown in the figure. How far apart the joists may be placed so that the bending stress in the joists does not exceed 8 N/mm^2.

Let x (in meters) be the centre to centre spacing of the joists. The area supported by one joist is { x/2 + x/2 } . 5 = 5x m^2

Total load supported by one interior joist = W = (25.x) kN

The maximum bending moment is given by M_{max} = (25 x . 10^3 . 5)/8 N m

The second moment of area of the joist = 1/12 . (150 . 300^3) = 3.375 . 10^8 mm^4

Now we have M = σ_{max} . Z = 8 . (3.375. 10^8)/[300/2] = 1.80 . 10^7 N mm

Thus (25x . 10^3 . 5)/8 . 10^3 = 15.625 x .10^6 =1.80 . 10^7 or x = 1.152 m

Example 4: A wooden beam is 80 mm wide and 120 mm deep with a semicircular groove of 20 mm radius cut out in the centre of each side. If the beam is simply supported over a span of 3 m and loaded with

(i) a concentrated load of 450 N at a distance of 1 m from left hand support
(ii) a uniformly distributed load of 500 N/m over the entire span of the beam

Calculate the maximum stress in the section.

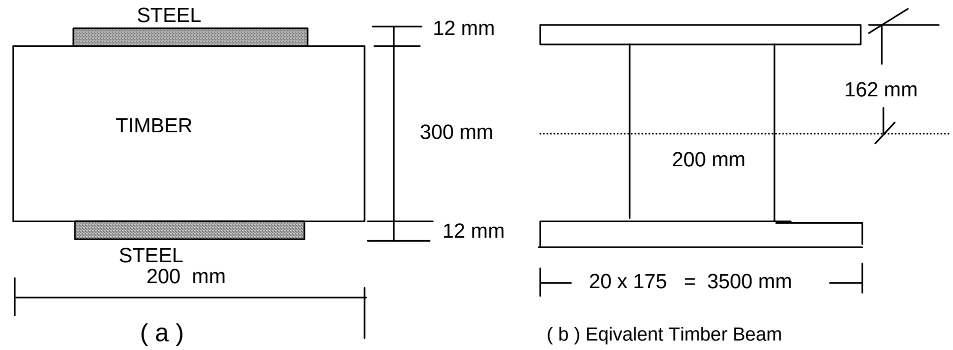
Let R_L and R_R be the reactions at the left hand the right hand supports. Thus
R_L = {500 x 3 x 3/2 + 450 x 2} / 3 = 1050 N
R_R = Total load – R_L = (500 x 3 + 450) – 1050 = 900 N
Shearing Force at a distance X from left hand support = 1050 – 450 –500.X
= 600 – 500X
For maximum bending moment S.F. = 0 thus 600-500X = 0 i.e X = 1.2 m from left hand support.
Thus maximum bending moment = 1050 x 1.2 – 450 x (1.2-1) – 500 x 1.2^2 /2
= 1260 – 90 – 360 = 810 N m

Maximum stress = M/ Z = {810 x 1000 x (120/2)} / I

I = 1/12 (80 x 120^3) – 2 {(1/2) [(πr^4)/4] } = 1.152x10^7 – 1.257x10^5 = 1.139x10^7
Thus
Maximum stress = (810x1000x60) / 1.139x10^7 = 4.267 N/mm^2

STRESS, STRAIN IN COMPOSITE BEAMS

Ex. 1: To calculate Moment of Resistance (Strength) of the composite beam. Assume allowable stresses in the materials not to exceed: σ_{st} = 140 N/mm^2 and σ_{ti} = 7 N/mm^2 and modular ratio m = (E_{st}/E_{ti}) = 20.



(a)   (b) Eqivalent Timber Beam

Equivalent Timber Beam

Using sketch (b)

$I_{xx} = 200 \times 300^3/12 + 3500\,[(324)^3 - (300)^3]/12 = 2.495X10^9$ mm^4

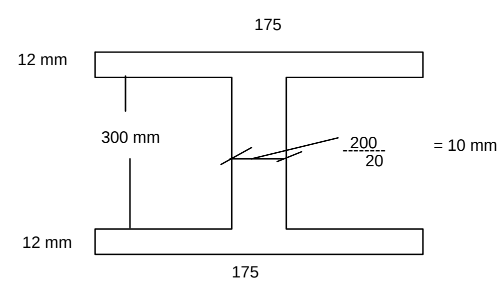
Moment of Resistance = $\sigma_{ti} \cdot Z = \{7 \times 2.495 \times 10^9 \times 10^{-6}\}/162$
= 107.819 kNm

Equivalent Steel Beam

$I_{xx} = 1/12 \cdot 175 \cdot (324)^3$
$- 1/12(175 - 10).(300)^3$
$= 1.2476X10^8$ mm^4

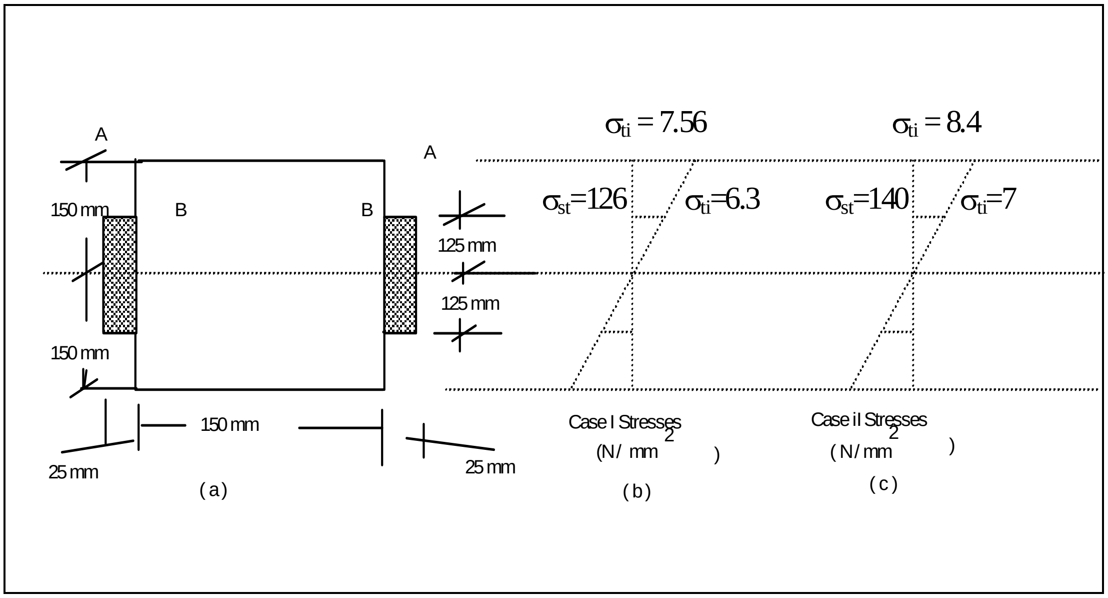
$M_R = \{140 \times 1.2476X10^8\}/162$
= 107.819 kN m

(Same as above)



EQUIVALENT STEEL BEAM

Ex. 2 Calculate Moment of Resistance of the beam if permissible stresses in the two materials are : σ_{st} = 126 N/mm^2 and σ_{ti} = 8.4 N/mm^2; and modular ratio, m = 20



Starting with Steel
<u>Case I Stresses</u> See sketch (b)

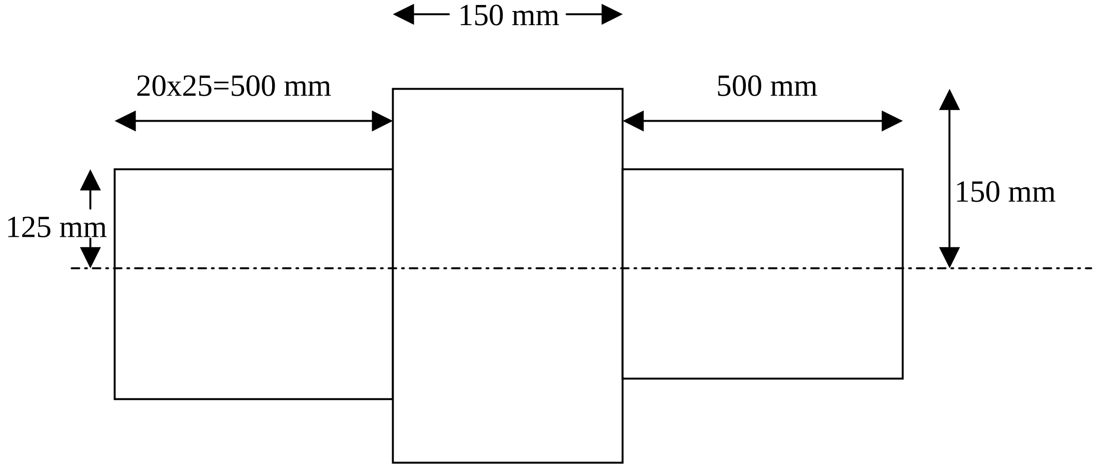
Let σ_{st} (at B-B) = 126 N/mm^2 then σ_{ti} (at B-B) = 126/20 = 6.3 N/mm^2
and σ_{ti} (at A-A) = 6.3 x 150/125 = 7.56 N/mm^2 both stresses are safe as less than permissible values[σ_{st} ; σ_{ti} = 8.4 N/mm^2].

Start with Timber
<u>Case II Stresses</u> See sketch (c)

Let σ_{ti} (at A-A) = 8.4 N/mm^2 then σ_{ti} (at B-B) = 8.4 x 125/150
= 7 N/mm^2
and σ_{st} (at B-B) = 20 x 7 = 140 N/mm^2, But permissible stress in steel is 126 N/mm^2 , hence σ_{st} is not safe.

Hence Case I stresses must be used.

Use an Equivalent Timber beam as shown



I_{xx} = 1000 x 250^3/12 + 150 x 300^3/12 = 1.6395 X 10^9 mm^4

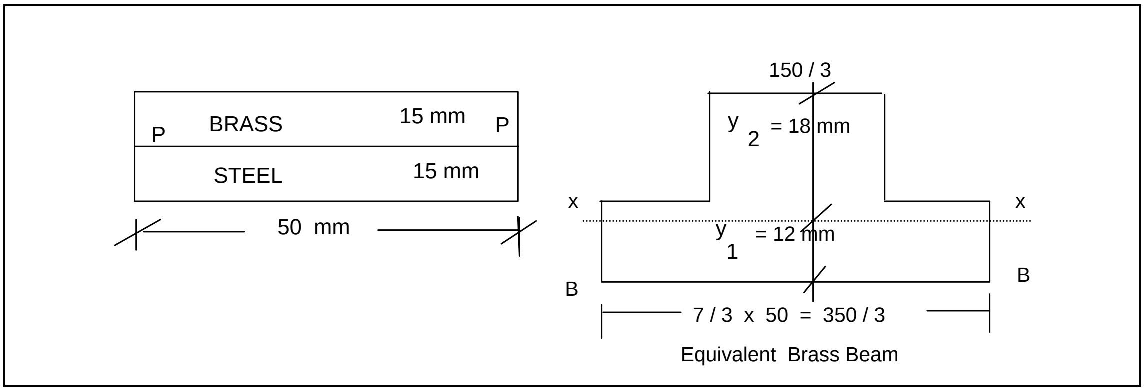
σ_{ti} at A-A = 7.56 N/mm^2
Since [M = σ I/y_{max}]

Thus M_R = {7.56 x 1.6395 X 10^9}/150 = 8.2635 X 10^7 N mm
= 82.635 kN m

If the beam is simply supported over a span of 3.6 m and carries a UDL of w/meter, then

M_R = $wl^2/8$ or w = 8 M_R/l^2 = (8 x 82.635)/$(3.6)^2$ = 51 kN/m (say)

Ex. 3 Given σ_{st} = 105 N/mm^2; E_{st} = 210 X 10^3; σ_{br}= 70 N/mm^2; E_{br} = 90 X 10^3 N/mm^2



At a plane P-P both materials have same strain; thus $\varepsilon_{pp} = \sigma_{st}/E_{st} = \sigma_{br}/E_{br}$
Thus $\sigma_{st} = \sigma_{br} . E_{st}/E_{br} = \sigma_{br}$. 210/90 = σ_{br} . 7/3; since m = Est / Ebr = 7/3; thus bottom flange of the eqivalent brass beam = 50 . 7/3 = 350/3 mm. To locate neutral axis take moments about B-B

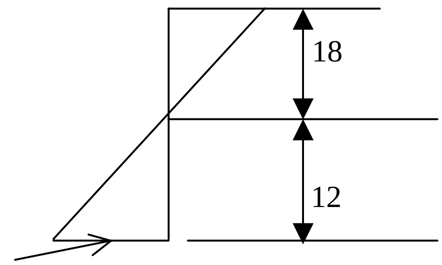
(350/3 x 15) x 15/2 + (150/3 x 15) x (15 + 15/2) = {350/3 x 15 + 150/3 x 15} y_1

Therefore, y_1 = 30,000/2500 = 12 mm

I_{xx} = 350/3 x 1/12 x $(15)^3$ + (350/3 x 15) x $(12 - 15/2)^2$ + 150/3 x 1/12 x 15^3 + (150/3 x 15)(18 - $15/2)^2$
= 1.65 X 10^5 mm^4

The stress diagrams must be drawn to determine maximum allowable stresses

CASE I Stresses (Note $\sigma_{br} = p_{br}$; $\sigma_{st} = p_{st}$) $\sigma_{br} = 70$



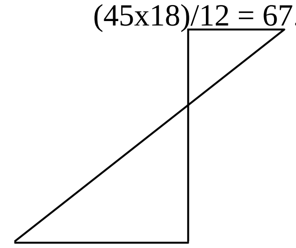
[70x12]/18

Let σ_{br} (top) = 70 N/mm^2

σ_{br} (bottom) = 70/18 x 12 N/mm^2
and σ_{st} (bottom) = 70/18 x 12 x 7/3
= 108.89 N/mm^2 against
105 N/mm^2 allowable(not safe)

CASE II Stresses

(45x18)/12 = 67.5



σ_{st} = 105 or σ_{br} = 105x3/7
= 45

Let σ_{st} (bottom) = 105 N/mm^2

σ_{br} (bottom) = 105 x 3/7 = 45 N/mm^2

and σ_{br} (top) = 45 x 18/12 = 67.5 N/mm^2
< 70 N/mm^2
(safe)

CASE II STRESSES ARE SAFE

Taking Equivalent brass beam and taking top stresses

M_R = 67.5 x 1.65 X 10^5/18 = 618750 N mm = 618.75 N m

or taking bottom stresses M_R = 45 x 1.65 X 10^5/12 = 618750
(same as above)

Stress Diagram for the composite Section

σ_{br} at P-P level = 67.5 x 3/18 = 11.25 N/mm^2

σ_{st} at P-P level = 11.25 x 7/3 = 26.25 N/mm^2

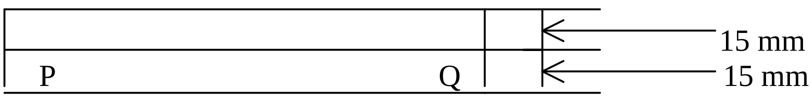
or
σ_{st} at P-P level = 3 x 105/12 = 26.25 N/mm^2

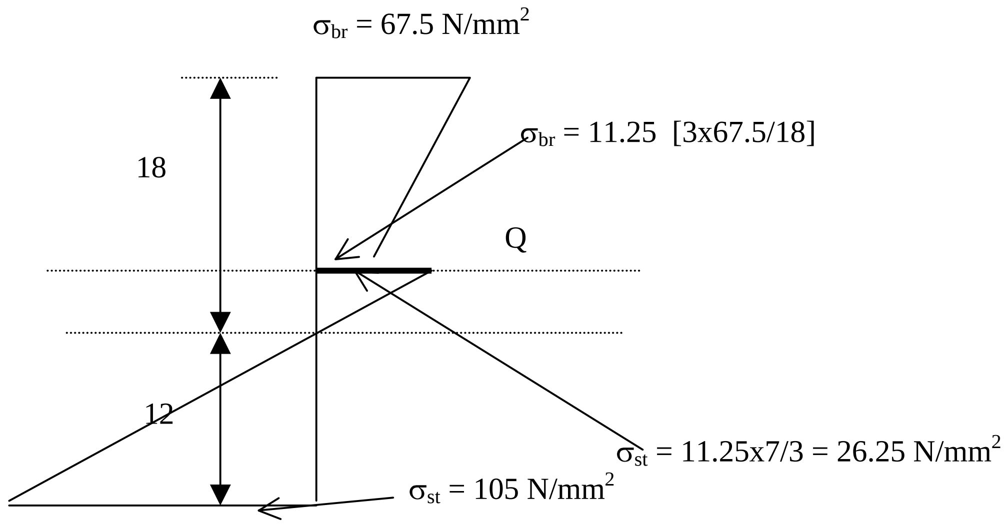
If the beam span is 1.2 m on a simple supports and carries a vertical concentrated load W at centre

$M_R = Wl/4$ or $W = 4 M_R/l$

Thus $W = 4 \times 618.75/1.2 = 2062.5$ N or 2.0625 kN.

STRESS DIAGRAM FOR COMPOSITE SECTION

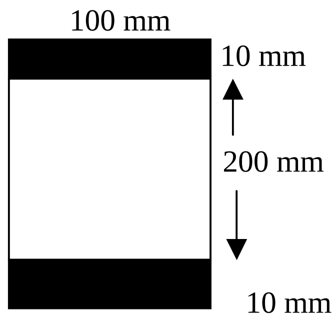




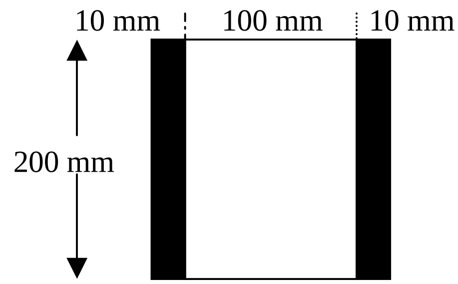
Example 4: (a) Explain, with sketches, the meaning of the term *Equivalent Timber Beam* with respect tp a compound beam of timber and steel. Hence, find an expression for the Moment of Resistance of the Equivalent Beam.

(b) A timber beam, 200 mm deep and 100 mm wide, is strengthened by the addition of steel plates rigidly fixed to it. The plates are 10 mm thick and may be fixed (a) right across the top and bottom of the timber beam or (b) the full length of the sides of the timber beam. If the modular ratio of steel to timber is 20, compare the strengths of the compound beams formed under the two conditions and find the Moment of Resistance of the stronger one when the allowable timber stress is 8 N/mm^2. If this beam carries a central concentrated load over a simply supported span of 3 m, what is the safe value of the load?

(b)



(I)



(II)

Using case I

$I_T = 100(200)^3 / 12 \quad = (8)\times10^8 / 12$

$$I_S = \frac{100}{12}\left[(220)^3 - (200)^3\right]$$

$= \{2.648\times10^8\}/12 \ \text{mm}^4$

$M_R = M_{RS} + M_{RT} = E_S I_S / R + E_T I_T / R$;

Hence $M_{RS} / M_{RT} = E_S I_S / E_T I_T = 20 \times 2.648/8 = 6.62$

Thus $M_R = 7.62\ M_{RT}$

Now using case II

$I_T = 8\times10^8/12$

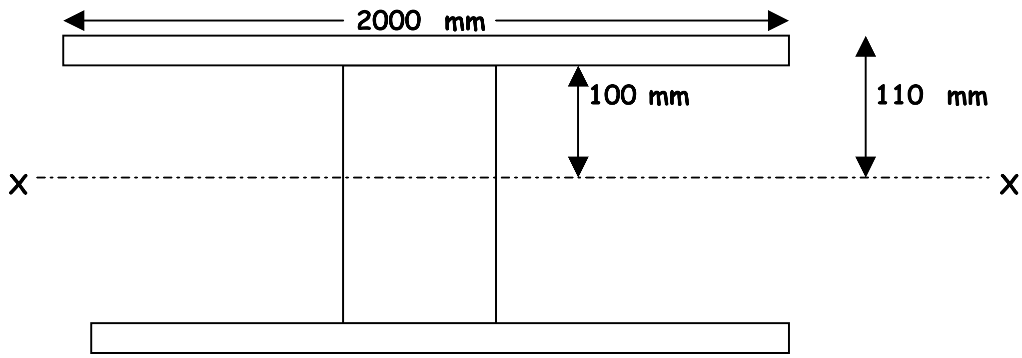
$I_S = 20 \times (200)^3 /12 = 1.6 \times 10^8 /12 \ \text{mm}^4$

$M_{RS} / M_{RT} = 20 \times 1.6 / 8 \quad = 4.0$ and $M_{RS} = 4\ M_{RT}$

Thus $M_R = 5\ M_{RT}$

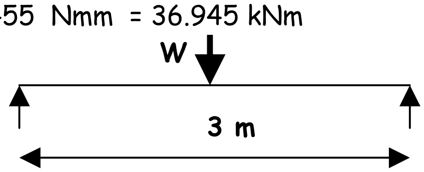
Ratio of Moment of Resistance and hence strength ratio = 7.62/5 = 1.524

Taking into consideration the Stronger Beam



I_{XX} = $8x10^8$ / 12 + 2000/12 [$(200)^3$ - $(200)^3$] = 5.08 x 10^8 mm^4

M_R = { $8x5.08x10^8$ }/ 110 = 36945455 Nmm = 36.945 kNm

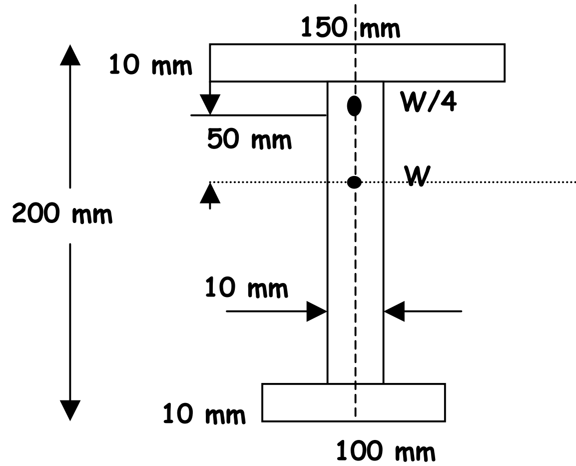


M_R = M_{max} = wl/4

Or W = 4 M_R / l = 4x36.945/3 = 49.26 kN

COMBINED BENDING AND AXIAL STRESS

Example 1. Given an I section asymmetrically loaded as shown in the figure. If the maximum compressive stresses allowed is not to exceed 8o N/mm^2. Calculate W which can be applied and also calculate the minimum stress in the section.



1. Locate C.G. Total Area of the section = 4300 mm^2

From top face, y_{top} x 4300 = (150 x 10 x 15) + 180 x 10 x (10 + 180/2) + 100 x 10 x (200 – 5) OR y_{top} = (382500)/(4300) = 88.95 mm

And y_{bottom} = 200 – 88.95 = 111.05 mm

2. calculate I_{xx}:

I_{xx} = [(150 x 10^3)/12 + 150x10x(88.95 – 10/2)^2] + [(10x180^3)/12 + 180x10 x(100 – 88.95)^2] + [(100x10^3)/12 + 100x10x(111.05 – 5)^2] = 2691.8624x10^4 mm^4

Total load = W + W/4 = 5W/4; Bending Moment due to eccentricity 50 mm for the load W/4 from the centroidal axis = 50 x W/4;

Section Modulus, Z= I/y = 2691.8624 x 10^4/88.95

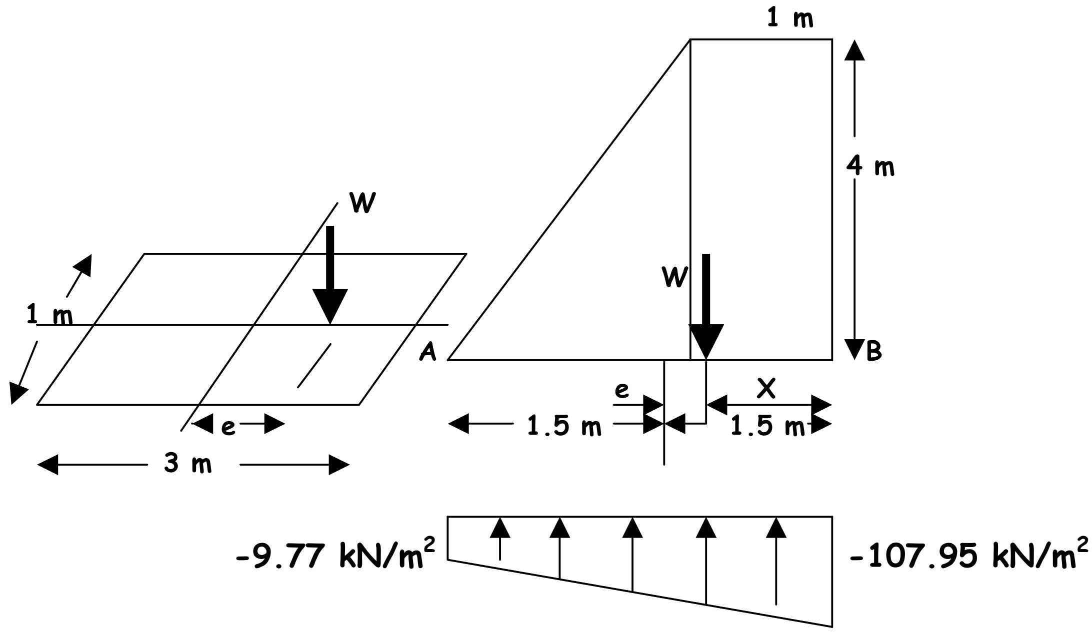
Allowable σ_{max} = - 80 N/mm^2 = -5W/(4x4300) – { (50 W/4) / [(2691.8624x10^4)/(88.95)]

Gives W = 240961.85 N OR 240.962 kN

σ_{min} = - (5x240962)/(4x4300) + (50x240962x111.05)/(4x2691.8624x10^4)

= - 57.621 N/mm^2 (compressive)

Example 2: Figure shows a retaining wall. The wall material has a density equal to 2250 Kg/m^3
Calculate the earth pressure developed at the base of the wall.



Pressure Diagram

Density of wall material = (2250x9.81)/1000 = 22.0725 kN/m^3

Weight W acts at a distance X from vertical wall
Hence [4 + (1+3)/2] . X = 1 x 4 x 0.5 + { (2x4)/2 + 2/3 + 1 }
OR X = 26/24 = 1.083 m
Hence eccentricity (e) of the load from centroid = 1.5 – 1.083 = 0.417 (which is less than d/6 (3/6 = 0.5 m) therefore the pressure is wholly compressive.

For Unit length of the wall, W = Area x 22.0725 = 8 x 22.9725 = 176.58 kN
[note Section Modulus, Z = 1/6 x (bd^2)
Hence stresses: σ_B = -176.58/3 – (176.58 x 0.417 x 6) / [1 x (3)^2]= -107.95 kN/m^2
σ_A = - 176.58/3 + (176.58 x 0.417 x 6) / [1x(3)^2] = - 9.77 kN/m^2

Example 3: Concrete column of length 4 m is subjected to a horizontal load of 12 kN and a vertical load of 450 kN at its top. If the section of the column is: depth = 500 mm and width = 300 mm and assuming that the vertical load acts through the centroid of the column section, calculate the maximum and the minimum stresses at the base of the column section. Neglect the weight of the column.

Vertical load of 450 kN at its top. If the section of the column is: depth = 500 mm and width = 300 mm and assuming that the vertical load acts through the centroid of the column section, calculate the maximum and the minimum stresses at the base of the column section. Neglect the weight of the column.

Vertical load, W = 450 kN; Maximum moment in the column at the base =W x 4
= 48 kN m

σ_{Base} = - W/A $\pm$ M/Z = - $(450 \times 10^3)/(300 \times 500) \pm 48 \times 10^6$ / {1/6 x 300 x 500^2}

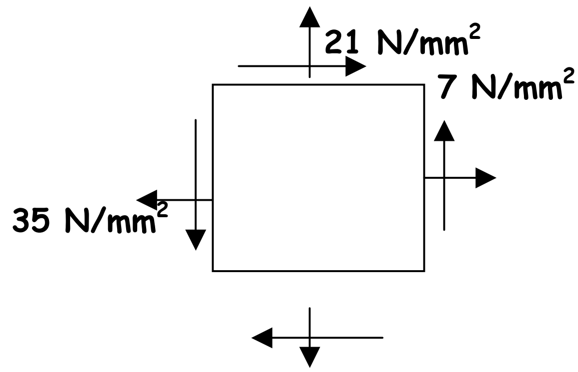
= -3 $\pm$ 3.84; thus σ_{max} = -6.84 N/mm^2 (compressive)

σ_{min} = + 0.84 N/mm^2 (tensile)

STRESS, STRAIN IN 3D BULK MASS SOLIDS

Example 1:

An element is subjected to tensile stresses σ_x = 35 MPa and σ_y = 21 MPa and a shearing strain τ = 7 MPa as shown in the figure. Determine principal stresses and the inclination of the principal plane.



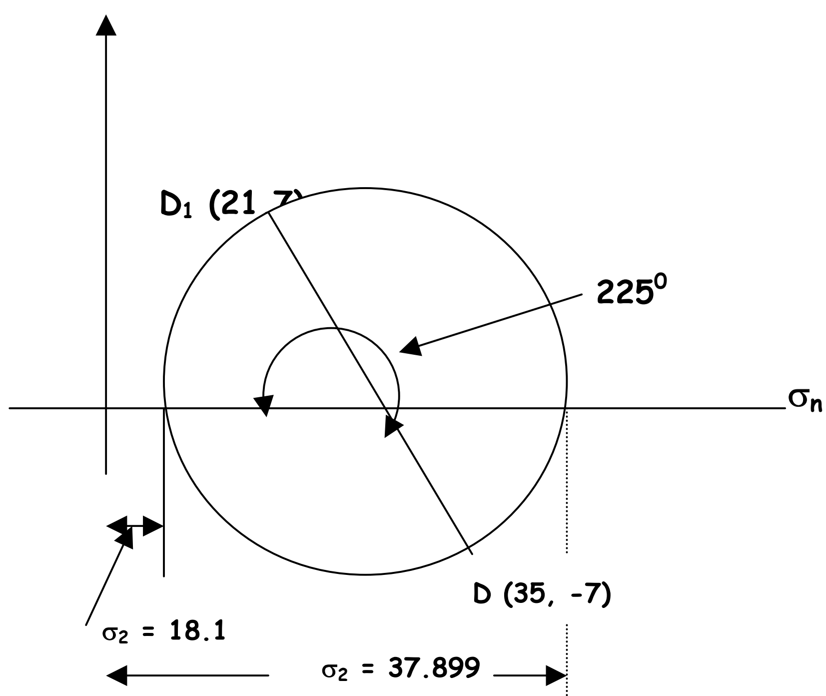
Since $\sigma_1\sigma_2 = 1/2\ (\sigma_x + \sigma_y) \pm 1/2\ \sqrt{\{(\sigma_x - \sigma_y)^2 + 4\ \tau_{xy}^2\}}$

$= 1/2\ (35+21) \pm 1/2\ \sqrt{\{(35-21)^2 + 4x(7)^2\}}$

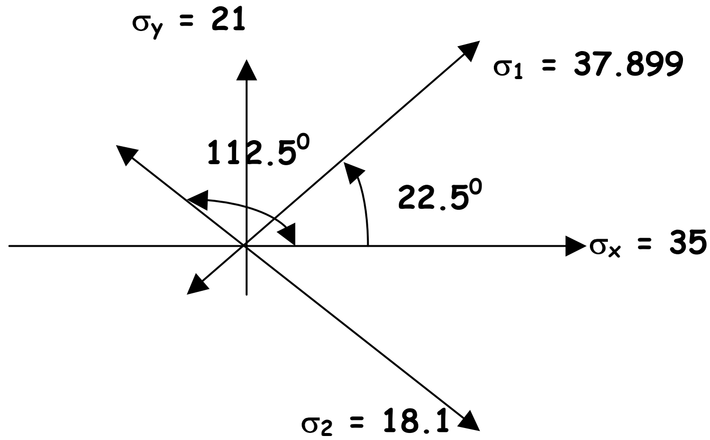
Thus $\sigma_1 = 28 + 9.899 = 37.899$ N/mm^2 and $\sigma_2 = 28 - 9.899 = 18.100$ N/mm^2

Tan $2\varphi = 2\ \tau_{xy} / (\sigma_x - \sigma_y) = 2x7/(35-21) = 1$

$2\varphi = 45^0$, 225^0 or $\varphi = 22.5^0$ and 112.5^0



On the element in the material



Example 2

A point is subjected to a tensile stress 60 N/mm^2 and a compressive stress of 40 N/mm^2, acting on two mutually perpendicular planes, and a shear stress of 10 N/mm^2 on these planes (down to the left and up to the right in the vertical planes). Determine the principal stresses as well as maximum shear stress. Also calculate the direction of the principal stresses. Show the entire stress system on a properly scaled diagram of Mohr's stress circle. Show also the directions of principal stresses on a sketch of the stress axes..

Using the established relationship for principal stresses

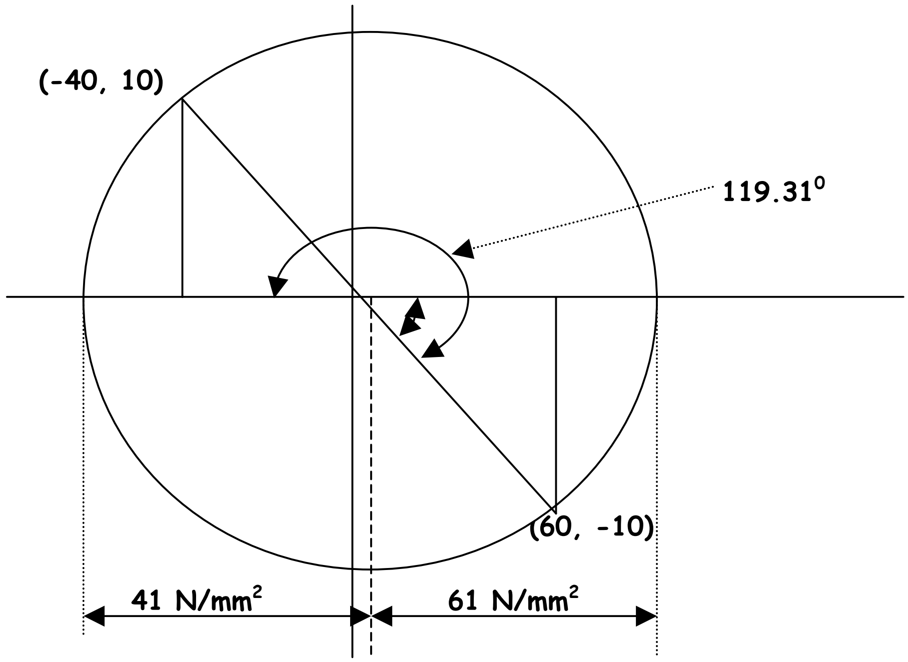
$\sigma_1, \sigma_2 = (60 - 40)/2 \pm \frac{1}{2}\sqrt{\{(60+40)^2 + 4\,(10)^2\}} = 10 \pm \frac{1}{2}\sqrt{\{10400\}} = 10 \pm 51$

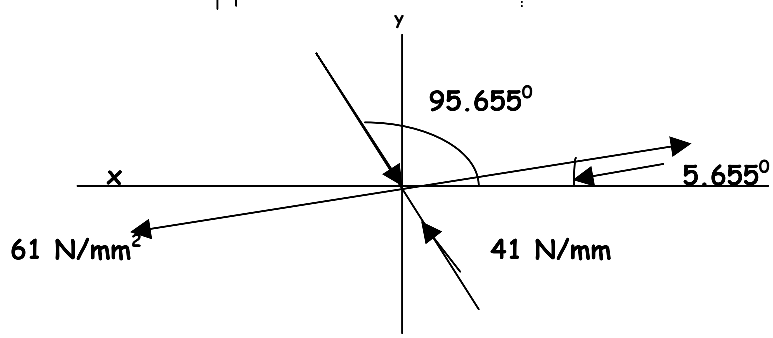
$\sigma_1 = 61$ N/mm^2 or $\sigma_2 = -41$ N/mm^2 (compressive)

$\tan 2\varphi = 2 \times (10) / 100 = 0.2$

$2\varphi = 11.31$ or 191.31 or $\varphi = 5.655^0$ or 95.655^0

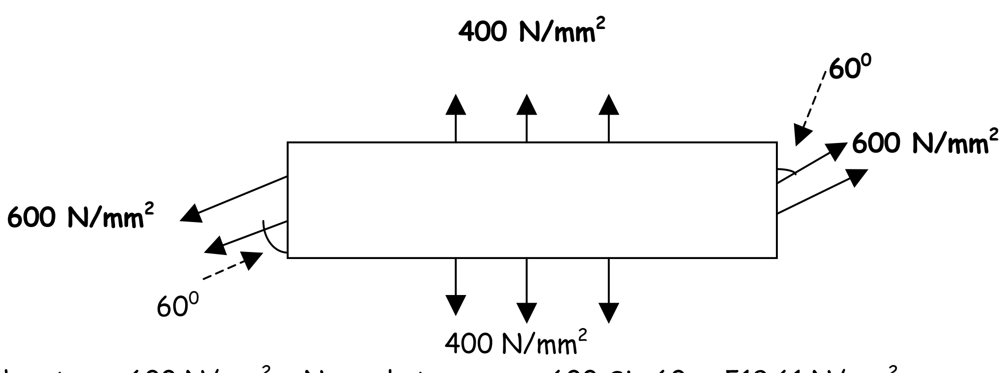
Maximum Shearing stress = $(\sigma_1 - \sigma_2)/2 = (61+41)/2 = 51$ N/mm^2





Example 3

A point in a strained material is subjected to the stresses shown in the figure. Locate the principal planes and evaluate the principal stresses.



Resolving the stress 600 N/mm^2 : Normal stress, σ_1 = 600 Sin 60 = 519.61 N/mm^2
Shearing stress = 600 Cos 60 = 300 N/mm^2
Stress on the perpendicular face = 400 N/mm^2
Since shearing stress is down to the left and up to the right it is +ve
Let ϕ be the inclination of the principal plane

Tan 2ϕ = (2 x 300) / (519.61 – 400) = 5.01630298 or $2\phi = 78.73^0$

Or ϕ = 39.365^0 or 129.365^0

Now principal stresses
σ_1 , σ_2 = (519.61 + 400)/2 ± ½√{(519.61 – 400)^2 + 4 x (300)^2}
or σ_1 = 459.805 + 305.903 = 756.708 N/mm^2 and σ_2 = 459.805 – 305.903 = 153.902 N/mm^2

Example 4
An element in a two-dimensional system is subjected to a tensile stress (σ_x) of 64 N/mm^2 and a compressive stress (σ_y) of 48 N/mm^2. If the major principal stress is limited to 80 N/mm^2, calculate the shear stress τ_{xy} , the maximum shear stress and the minor principal stress.
Calculate also the directions of the principal and shear stresses.
Illustrate the results on a scaled diagram of Mohr's circle and show the position and direction of the principal and maximum shear stresses on a sketch of an element.

Using the derived results

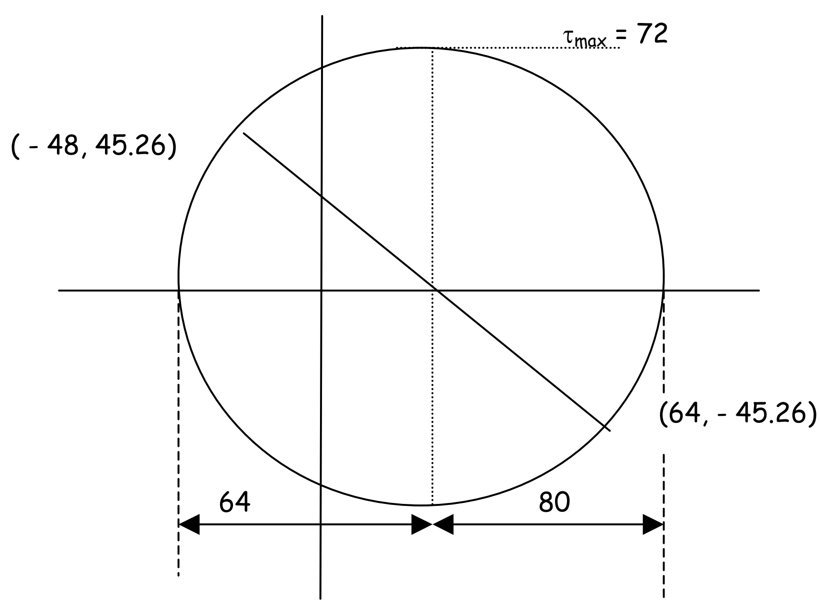
σ_1 = (64-48)/2 ±½ √ {(64 + 48)^2 + 4 (τ_{xy})^2 } or τ_{xy} = 45.26 N/mm^2

Now τ_{max} = ½√ { (σ_x -σ_y)^2 + 4 (τ_{xy})^2 } = ½√ { (64+48)^2 + 4 (45.26)^2 } = 72 N/mm^2

Also τ_{max} = ½ (σ_1 -σ_2) or σ_2 = σ_1 - 2 τ_{max} = 80 – 2 (72) = - 64 N/mm^2

Tan 2 φ = (2 x 45.26) / (64 + 48) thus $\varphi = 19.47^0$ or 109.470 direction of principal stresses

And direction of shear stress = $\varphi + 45^0 = 64.47^0$

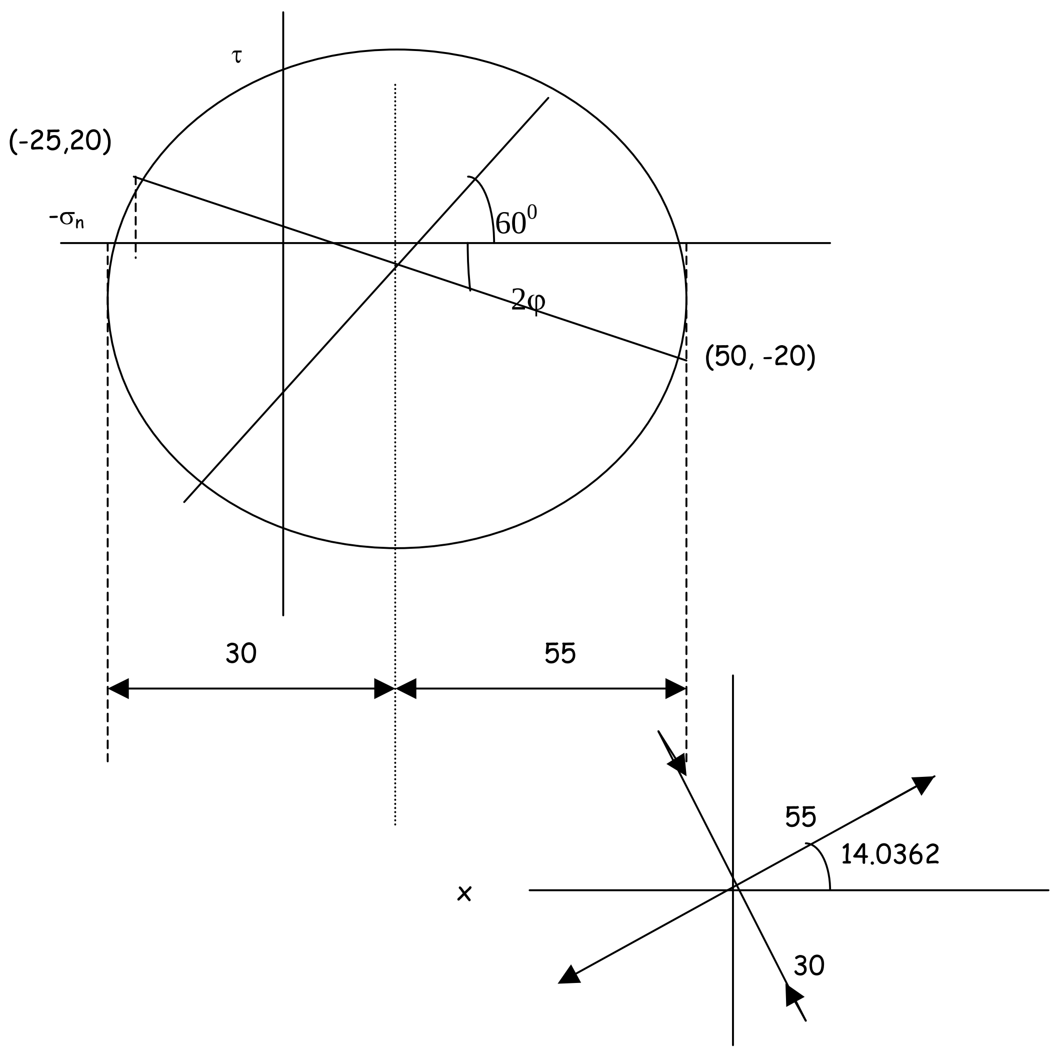


Example 5

An element of a vertical plane of a vertical plane is subjected to bi-axial stresses of 50 N/mm^2 (tensile) and 25 N/mm^2 (compressive) in the horizontal and vertical directions respectively. It is also subjected to a shear stress of 20 N/mm^2 down to the left and up to the right in a vertical direction.
Calculate the principal and maximum shear stresses and determine the direction of the major principal stress. Show these clearly on a properly dimensioned and scaled diagram of Mohr's stress circle. Show also the principal stresses and directions on a sketch of the axes. Determine also the normal and shear stresses on a plane whose normal makes an angle of 30^0 with the direction of the major principal stress.

σ_1 and σ_2 = $(50-25)/2 \pm \frac{1}{2}\sqrt{\{(50+25)^2 + 4(20)^2\}}$ = 12.5 ± 42.5 or σ_1 = 55 N/mm^2 and σ_2 = - 30 N/mm^2 and τ_{max} = 42.5 N/mm^2

tan 2φ = (2x20)/75 0.5333 or 2φ = 28.0725^0 or φ = 14.0362^0



For $\theta = (14.0362 + 30)^0$

$\sigma_n = \frac{1}{2}(\sigma_x - \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\ Cos\ 2\theta\ + \tau_{xy}\ Sin\ 2\theta$

$= (50 - 25)/2 + (50 + 25)/2\ Cos\ 88.0724^0 + 20\ Sin\ 88.0724^0 = 33.7502\ N/mm^2$

Similarly $\tau = \frac{1}{2}(\sigma_x - \sigma_y)\ Sin\ 2\theta - \tau_{xy}\ Cos\ 2\theta$

$= (50 + 25)/2\ Sin\ 88.0724^0 - 20\ Cos\ 88.0724^0$

$= 36.8058\ N/mm^2$

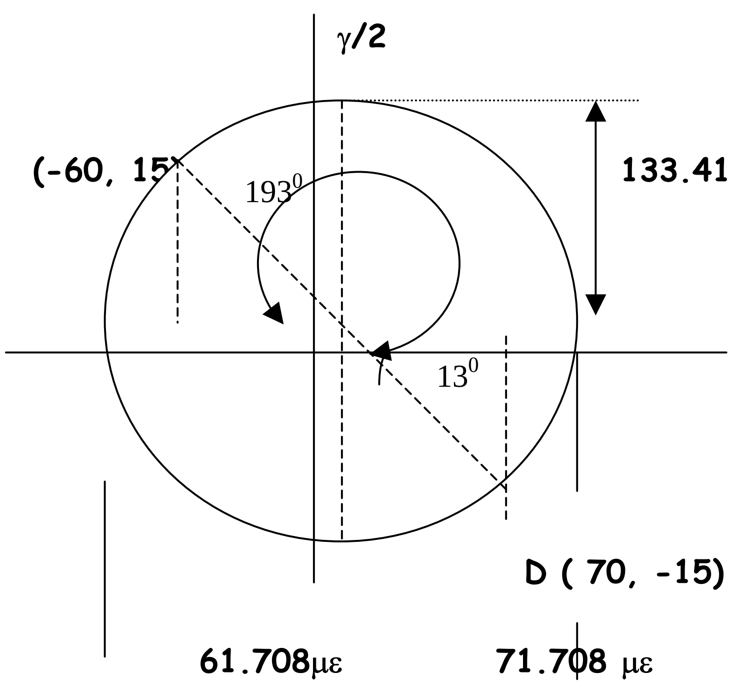
Example 1

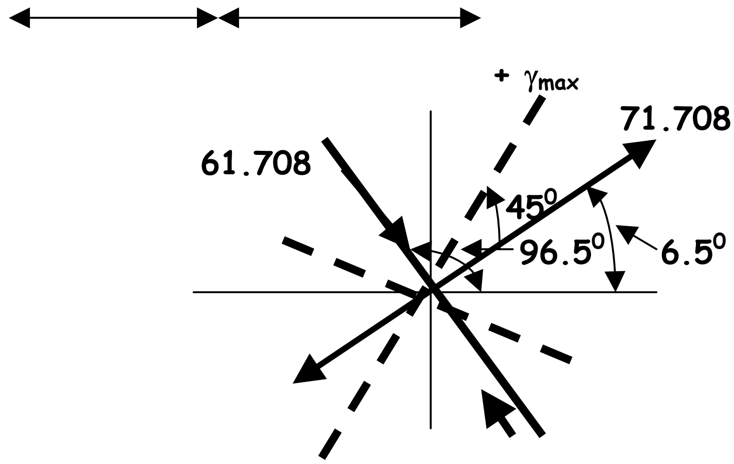
An element of material is strained such that $\varepsilon_x = 70$ microstrain ; $\varepsilon_y = -60$ microstrain and $\gamma_{xy} = 30$ micristrain. Calculate principal strains ε_1, ε_2 ,γ_{max} . Represent the results on Mohr's circle of strain and on the element

$\varepsilon_{1,}\varepsilon_2 = [\varepsilon_x + \varepsilon_y]/2 \pm \frac{1}{2}\sqrt{\{(\varepsilon_x - \varepsilon_y)^2 + (\gamma_{xy})^2\}} = (70 - 60)/2 \pm \frac{1}{2}\sqrt{\{(70+60)^2 + 30^2\}}$

thus $\varepsilon_1 = 71.708\ \mu\varepsilon$ (tensile) and $\varepsilon_2 = -61.708\ \mu\varepsilon$ (compressive)
$\gamma_{xy} = \pm(\varepsilon_1 - \varepsilon_2) = (71.708 + 61.708) = \pm 133.416\ \mu\varepsilon$

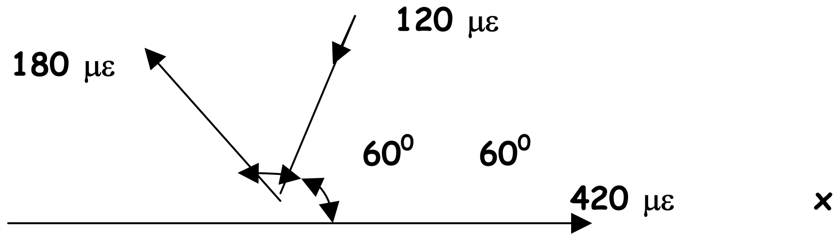
$\tan 2\phi = \gamma_{xy} / (\varepsilon_x - \varepsilon_y) = 30/(70+60) = +0.230769$
thus $2\phi = 13^0$ and 193^0 and $\phi = 6.5^0$ and 96.5^0





Example 2

A Strain rosette placed at a point on an element of stressed material gives outputs of 420, -120 and 180 $\mu\varepsilon$ in directions 60^0 apart respectively. Calculate the principal strains and their direction. Show them on a diagram of Mohr's Strain Circle drawn to scale and sketch them on a bi-axial strain diagram. Determine the maximumshear strain.



Using the derived relationship

$\varepsilon_n = (\varepsilon_x + \varepsilon_y)/2 +$ 🗁 $(\varepsilon_x - \varepsilon_y)$ Cos $2\theta + \gamma_{xy}/2$ Sin 2θ

ε_n (at $\theta = 0$) = $\varepsilon_x = 420$ ……………………………………………………………………………………(1)

ε_n (at $\theta = 60$) = $(\varepsilon_x + \varepsilon_y)/2 + (\varepsilon_x - \varepsilon_y)/2 \,.\, (-1/2) + (\gamma_{xy})/2 \times (\sqrt{3}/2) = -120$

or $\varepsilon_x + 3\varepsilon_y + \sqrt{3}\,\gamma_{xy} = 720$ ……………………………………………………………………………………(2)

ε_n (at $\theta = 120$) = $(\varepsilon_x + \varepsilon_y)/2 + (\varepsilon_x - \varepsilon_y)/2$ Cos 240 $+ (\gamma_{xy})/2$ Sin 240 = 180

or $\varepsilon_x + 3\varepsilon_y - \sqrt{3}\,\gamma_{xy} = 720$ ……………………………………………………………………………………(3)

Solving above equations, we get

ε_x = 420 $\mu\varepsilon$; ε_y = - 100 $\mu\varepsilon$ and γ_{xy} = -346.41 $\mu\varepsilon$

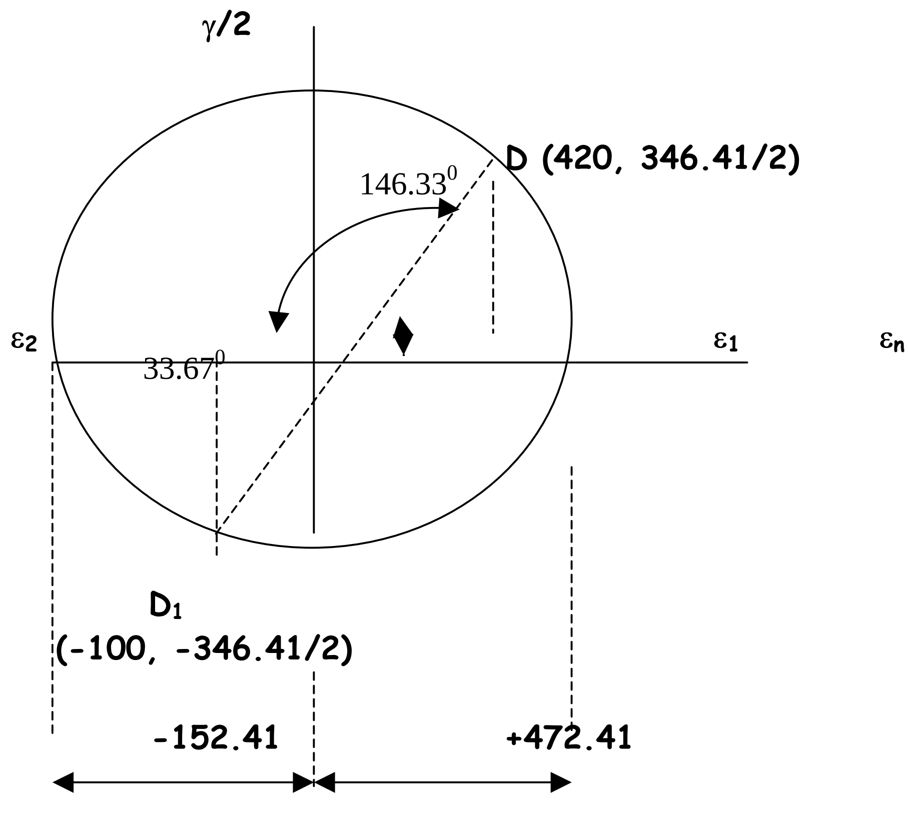
Principal Strains

Using the derived relationship

ε_1 = (420 -100)/2 ± 🗁✰ { $(420 + 100)^2 + (-346.41)^2$ }

= 160 ± 312.41

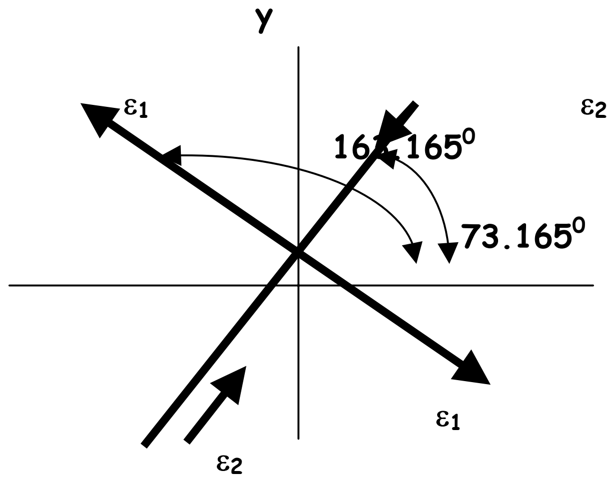
Thus ε_1 = 472.41 $\mu\varepsilon$; ε_2 = - 152.41 $\mu\varepsilon$ and γ_{max} =± (472.41 + 152.41) = ± 624.82 $\mu\varepsilon$



$\tan 2\phi = \gamma_{xy} / (\varepsilon_x - \varepsilon_y)$ = -346.41 / (420 + 100) = - 0.666173

thus 2ϕ = - 33.67^0 or $2\phi = 146.33^0$ or 326.33^0

$\phi = 73.165^0$ or 163.165^0



y
ε1
ε2
163.165⁰
73.165⁰
ε1
ε2