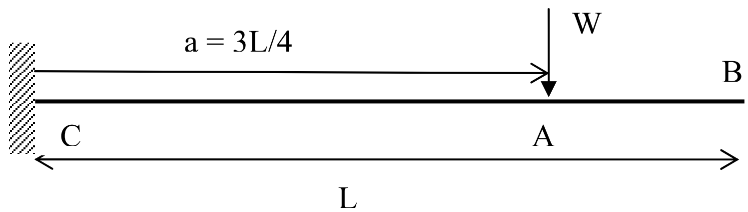
MECHANICS OF SOLIDS TUTORIAL SOLUTIONS 2005

R. Clarke

2. Practice Sheet 6 Question 1

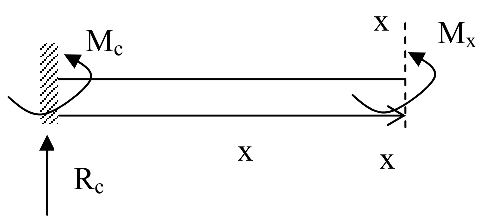


a. RTS slope at A $= 9WL^2/32EI$

b. RTS (deflection at B/deflection at A) = 1.5

a.

Taking a section between CA,



A cantilever has a moment reaction and vertical reaction at the support. These are determined by taking moments. Hence,

$M_c = Wa$ and $R_c = W$

With the sign convention that sagging moments are +ve, and deflection upwards is +ve, and for equilibrium, $\Sigma M = 0$

$M_x - Wx + Wa = 0$
$M_x = Wx - Wa$

$EI\,(d^2y/dx^2) = M_x = Wx - Wa$
$EI\,(dy/dx) = Wx^2/2 - Wax + P$ (1)

At x = 0 dy/dx = 0 hence P = 0
Hence slope = dy/dx = $(Wx^2/2 – Wax)/EI$

At point A, a = 3L/4 = x
hence slope at A = $(Wx^2/2 – Wx^2)/EI = -Wa^2/2EI$
Substituting a = 3L/4,
slope at A = $-W3^2L^2/(4^2 \text{ x } 2EI) = -9WL^2/32EI$

b.

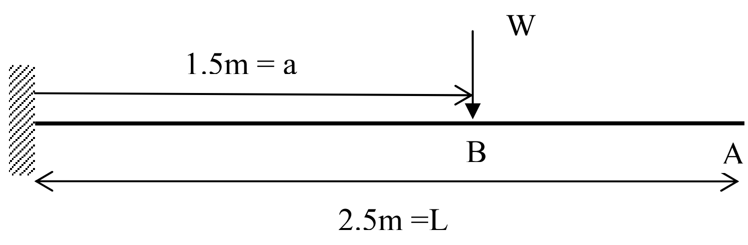
From (1),
EI y = $Wx^3/6 – Wax^2/2 + Px + Q$
At x = 0 y = 0 hence Q = 0; P = 0 from earlier.
Hence y = deflection = $(Wx^3/6 – Wax^2/2)/EI$
For the deflection at A, a = 3L/4 = x
Hence deflection at A = $(Wx^3/6 – Wx^3/2)/EI = -Wa^3/3EI = -W(3L/4)^3/3EI$
= $-27WL^3/192EI$

Now, the deflection at B = (slope at A x L/4) + deflection at A
hence (deflection at B/deflection at A) = (slope at A x L/(4x deflection at A)) + 1

From part "a" of the question,
deflection at B/deflection at A = $((-9WL^2/32EI)$ x L/(4x deflection at A)) + 1
= ((9/32) x (192/(4x27))) + 1
= 1.5

3. Practice Sheet 8 Question 4

a.



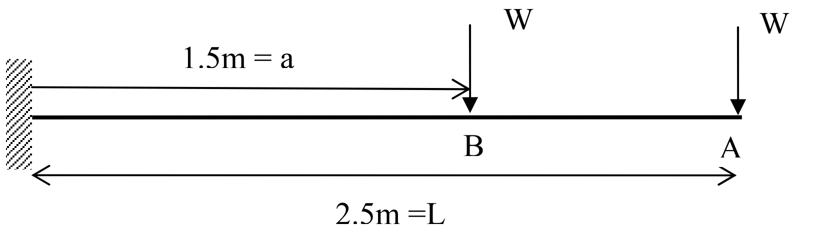
RTF deflection at A.

Deflection at A = slope at B x (L-a) + deflection at B

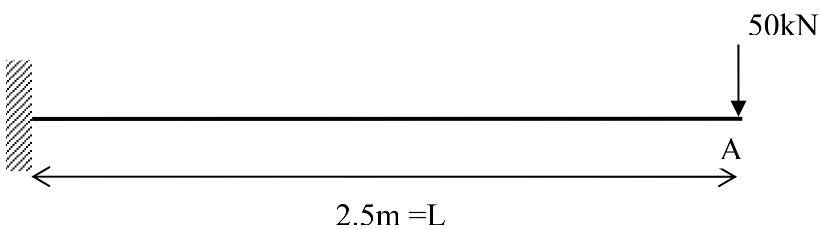
From the standard cases,

Slope at B = $Wa^2/2EI$ and Deflection at B = $Wa^3/3EI$
Hence deflection at A = $((W(1.5)^2/2EI) \times (2.5-1.5)) + W(1.5)^3/3EI$
$= (W/EI)(1.5^2 \times ½ + 1.5^3/3) = 2.25W/EI$

b.



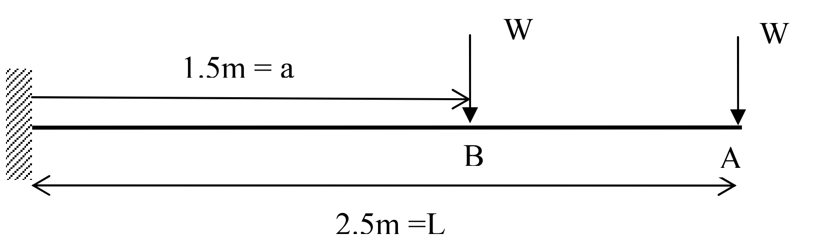
CASE I



CASE 2

RTF W such that the deflection at A for CASE 1 is the same as the deflection at A for CASE 2.

Using superposition for CASE 1,



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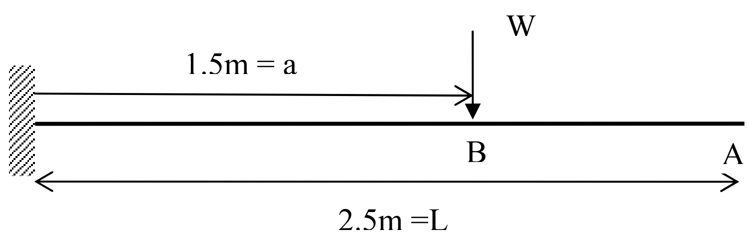

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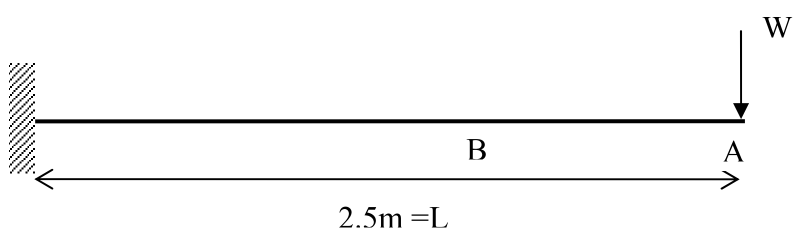


Hence using the results from part a,

CASE 1 deflection at A = $2.25W/EI + WL^3/3EI$
Sub. L = 2.5m and a = 1.5m
CASE 1 deflection at A = $2.25W/EI + (W(2.5)^3/3EI)$
$= (W/EI)\ (2.25 + 5.208) = 7.458W/EI$

CASE 2 deflection at A = $WL^3/3EI = 50 \times 2.5^3/3EI = 260.417/EI$

Equating the deflections for CASEs 1and 2,

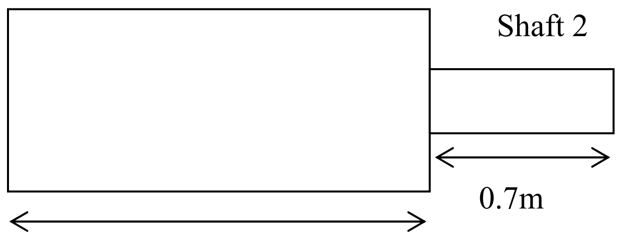
7.458W = 260.417
W = 34.9 kN

c.

The maximum stresses are at the beam section at the support, and the internal resisting moment must equal the applied moment at a section.
$\sigma_{1,R} = M_{1,R} / Z$
$\sigma_{2,R} = M_{2,R} / Z$
$\sigma_1 / \sigma_2 = M_{1,R} / M_{2,R} = M_{1,\text{applied}} / M_{2,\text{applied}}$
Hence the ratio of the strengths is the ratio of the applied moments,

Applied moment due to 2-load case = 1.5W + 2.5W = 4 x 34.9 = 139.6 kNm
Applied moment due to 1-load case = 50 x 2.5 = 125
Hence strength ratio = 139.6/125 = 1.117

4. Practice Sheet 9 Question 1



RTF Power transmitted, P, if N = 500 rpm

Since the shafts are in series, the torque is the same in both shafts but the twist angles are different.

Hence as the materials are the same,

$I_{p1}\ \theta_1 / L_1 = I_{p2}\ \theta_2 / L_2$ or

$\theta_1 = (L_1/L_2)\ (I_{p2}/ I_{p1})\ \theta_2$ (1)

$\theta_2 = 2° - \theta_1 = 0.0349 - \theta_1$ (converting to radians)

$L_1/L_2 = 2$ and $I_{p2}/ I_{p1} = 50^4/100^4$

Sub. in (1),

$\theta_1 = 2 \text{ x } 50^4/100^4\ (0.0349 - \theta_1)$

$8\theta_1 = 0.0349 - \theta_1$

Hence $\theta_1 = 3.88 \text{ x } 10^{-3}$ rad

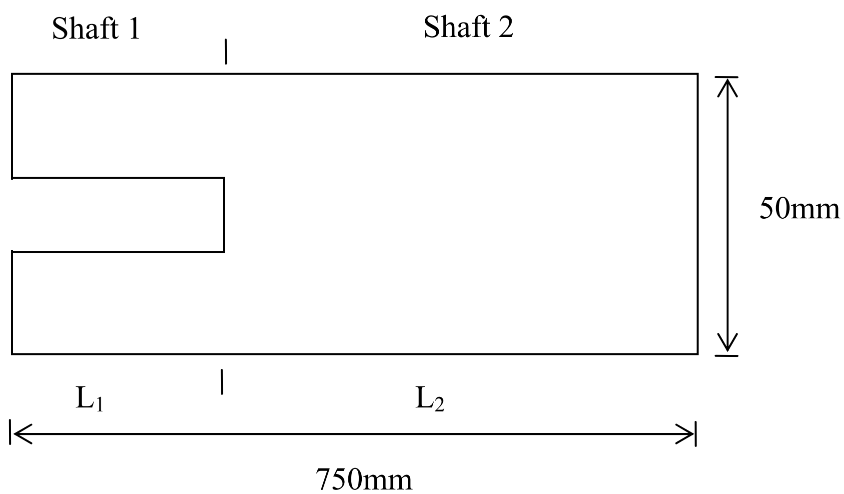
$T = G\ \theta_1\ I_{p1}/ L_1$

$= 3.14 \text{ x } 80 \text{ x } 10^3 \text{ x } 3.88 \text{ x } 10^{-3} \text{ x } 100^4 / (2 \text{ x } 1400 \text{ x } 10^3) = 3.48 \text{ x } 10^4$ Nm

$P = \pi NT/30$ Nm/s $= 3.14 \text{ x } 500 \text{ x } 3.48 \text{ x } 10^4 / 30 = 1821.2 \text{ x } 10^3$ Nm/s

= 1821.2 kNm/s

5. Practice Sheet 9 Question 3



a.

As the shafts are in series, the torque in the shafts is the same but the twist angles are different.

As the shafts are of the same material and outer radius,

$T = I_{p1}\ \tau_{01} = I_{p2}\ \tau_{02}$

$\tau_{01}/\tau_{02} = I_{p2}/\ I_{p1}$

Therefore, as the I_p for a solid shaft is greater than that for a hollow shaft of the same material and outer radius, the hollow shaft will be experiencing the higher shear stress, hence will be the "weaker" shaft.

Therefore, the inner radius of the hollow shaft is determined from

$T = I_{p1}\ \tau_{01}\ /\ r_0$

Substituting values,

$1.67 \text{ x } 10^6 = 0.5 \text{ x } 3.1415927 \text{ x } (25^4 - r_i^4) \text{ x } 75\ /25$

$25^4 - r_i^4 = 3.54385 \text{ x } 10^5$
$r_i = 13.8$mm
Inner diameter = 13.8 x 2 = 27.6 mm

b.

$\theta_1 + \theta_2 = 1.5° = 2.619 \times 10^{-2}$ (radians)

$\theta_1 = T L_1/G I_{p1}$; $\theta_2 = T L_2/G I_{p1}$
But $L_1 = 750 - L_2$ hence,

$$\frac{1.67\text{x}10^6\, L_1}{80\text{x}10^3\text{x}\pi/2(25^4 - 13.8^4)} + \frac{1.67\text{x}10^6\,(750\text{-}L_1)}{80\text{x}10^3\text{x}\pi/2(25^4)} = 2.619 \times 10^{-2}$$

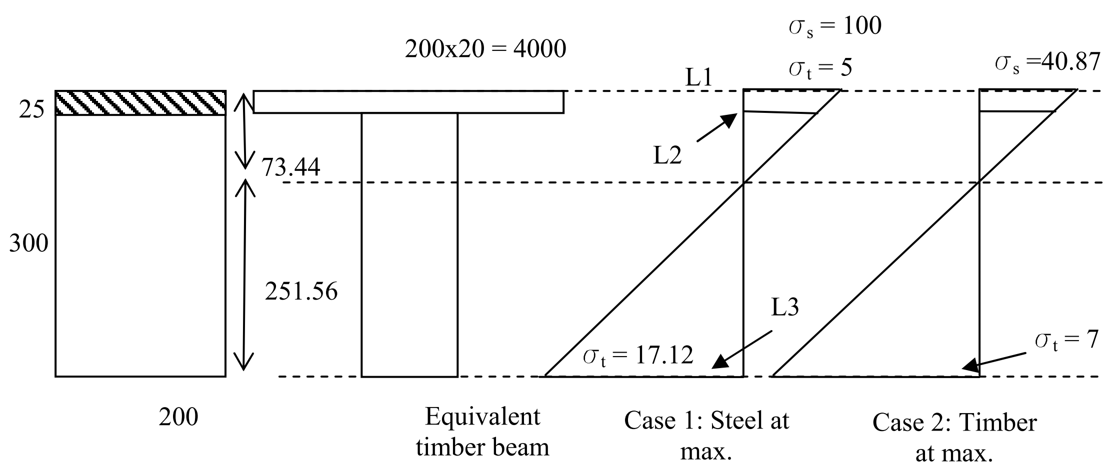
$$\frac{1.67\text{x}10^6\, L_1}{(25^4 - 13.8^4)} + \frac{1.67\text{x}10^6\,(750\text{-}L_1)}{(25^4)} = 2.619 \times 10^{-2} \times 80\text{x}10^3\text{x}\pi/2$$

$4.71275L_1 + 3206.4 - 4.2752L_1 = 3291.13$
$0.43755\, L_1 = 84.73$
$L_1 = 193.6$mm

5. Practice Sheet 9 Question 3

A 4m long composite beam is comprised of a steel plate 25mm deep x 200mm wide, on the top of a timber beam 300 mm deep x 200mm wide. Can this beam safely carry a uniformly distributed load of 15 kN/m if the allowable stresses in the steel and timber are 100 MPa and 7 MPa respectively, and the modular ratio (steel to timber) is 20?

As the distribution of material is not symmetrical, the neutral axis cannot be at the mid-depth of the beam. It is convenient to determine the maximum stresses in the materials using an equivalent beam.



Taking moments of area from the bottom of the equivalent timber beam,
((25x4000)+(300x200))y' = (300x200x150)+(25x4000x(300+12.5))

Horizontal centroidal axis distance, y'= 251.56mm from the bottom
I for equivalent timber beam = $4000 x 25^3/12 + 200 x 300^3/12 + 4000 x 25 x (73.44-12.5)^2 + 200 x 300 x (251.56-150)^2 = 14.4 \times 10^8$ mm^4

It must be checked that when one material is at its allowable stress, the stress in the other material is not higher than its allowable stress.

Case 1: Steel at its allowable stress = 100 MPa (at Level 1)
Hence at Level 1, the timber stress is = 100/20 = 5.0
Hence maximum timber stress is at Level 3 = 5 x 251.56/73.44 = 17.2 > 7 MPa : NOT SAFE

Case 2: Timber at its allowable stress = 7 MPa (at Level 3)
Hence at maximum steel stress is at Level 1 = 7 x 20 x 73.44/251.56 = 40.87 < 100 MPa
Therefore the Case 2 stress distribution is safe.

Moment of Resistance of Composite Beam:
$M_R = \sigma_{timber\ max}\ I_{eq.\ timber}\ /\ y_{timber\ max}$
$M_R = (7 x 14.4 x 10^8/251.56) x 10^{-6}$ kNm = 40.1 kNm
$M_{applied} = 15 \times 4^2 /8 = 30 < 40.1$ kNm
Hence the beam can safely carry the load.