11.8.5 Example Problem – Reinforced Masonry Shear Wall (Strength Design)

This example problem is a variation of example 3K on page 95 of the book entitled "Reinforced Masonry Engineering Handbook - Clay and Concrete Masonry" by James Amrhein(11-27). Determine if the CMU shear wall shown in Figure 11-27 is adequate for the following vertical and seismic loads. Use strength design UBC 97.

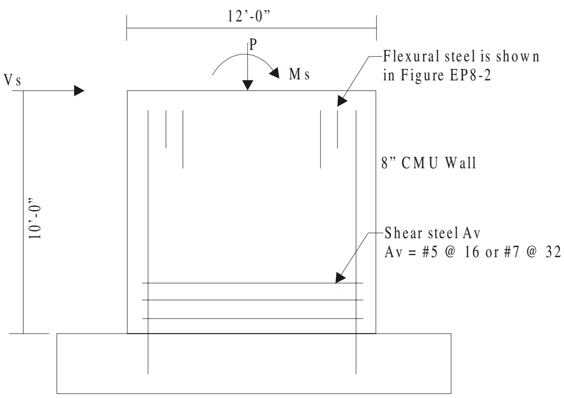

Figure 11-27. Elevation of Shear Wall

Loads: Dead Load= 30 kips
Live Load = 0 kips
Lateral Shear Force (V_E) = 75 kips
Seismic Moment (M_E) = 400 kip-ft

Load Factors: U = 1.2D + 1.6 L
U = 1.2D + 0.5L + 1.0E
U = 0.9D ± 1.0E

Reduction Factors: ϕ = 0.65 Axial
ϕ = 0.65 Axial plus flexure
ϕ = 0.80 Flexure only
ϕ = 0.60 Shear

Wall Properties:

Wall is fully grouted ($M_n \geq 1.8\ M_{cr}$)
Normal block thickness = 8 inch
Actual block thickness (b) = 7.625 inch
Length of wall (L) = 12 ft
Specified compressive strength(f_m') = 1500 psi
Modulus of rupture (f_r) = $4.0\sqrt{f_m'}$
Maximum usable masonry strain (e_{mu}) = 0.003
Modulus of elasticity of CMU (E_m) =750f_m'
Shear modulus of masonry (G) =0.4E_m
Specified yield strength of steel(f_y) =60 ksi
Modulus of elasticity of steel (E_s) = 29 x 10^6psi

<u>SOLUTION OUTLINE:</u>

A. Interaction diagram (generate/draw)
B. Cracking moment strength (M_{cr})
C. Load cases (axial plus flexure)
D. Boundary members
E. Shear

A. Interaction Diagram

1. Nominal axial load strength (P_o)

$P_o = 0.85\ f_m'(A_e - A_s) + f_yA_s$
= 0.85(1.5ksi)[12 ft (12 in/ft)(7.625 in) - 10 bars0.31 in^2/bar] + 60 ksi (10 bars) (0.31 in^2/bar)
= 1581.99 kips

2. Design axial load strength (P_u)

$P_u = \phi(0.80)(P_o)$
= 0.65(0.80)(1581.99 kips)
= 822.64 kips

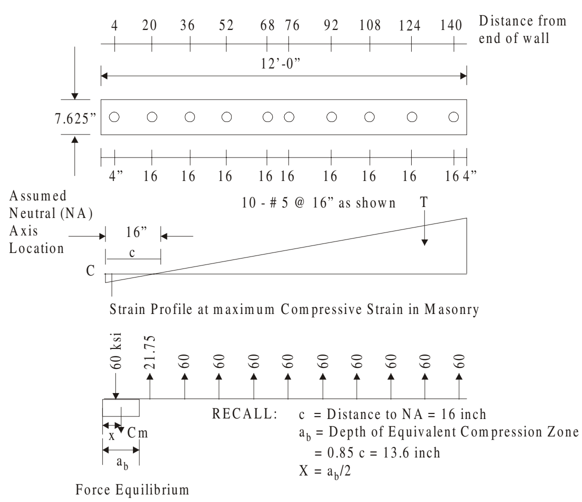

Figure 11-28. Steel locations, strain profile and force equilibrium diagrams

3. Nominal bending moment strength (M_o): See Figure 11-28.

Must solve for location of neutral axis (NA) such that sum of axial forces on cross section is zero.

_ Assume location for NA; c = 16 inch.

_Use maximum allowable CMU strain of 0.003.

_ Iterative solution.

_ Take sum of moments about extreme compression fiber (end of wall).

$T = A_s f_s = [21.75 \text{ ksi} + 8(60 \text{ ksi})](0.31 \text{ in}^2)$
$= 155$ kips

$C = A_s f_s + \phi f_m' b a_b$
$= 0.31\text{in}^2 (60 \text{ ksi}) + 0.85 (1.5 \text{ ksi})(7.625 \text{ in})(13.6 \text{ in})$
$= 150.82$ kips

T-C = 4 kips close enough use c = 16".

$M_o = A_s f_y - 0.85 f_m' b a_b$

$M_o = 0.31 \text{ in}^2[21.75(20) + 60(36 + 52 + 68 + 76 + 92 + 108 + 124 + 140] - 0.31 \text{ in}^2 \times(60)(4) - 0.85(1.5)(13.6)^2(1/2)(7.625)$
$= 13080.4 - 74.4 - 899$
$= 12{,}107$ k-in
$= 1009$ k-ft

4. Design bending moment strength (M_u)

$M_u = 0.80 M_o$
$= 0.80(1009 \text{ k-ft})$
$= 807.2$ k-ft

5. Nominal balanced design axial strength (P_b): See Figure 11-29.

$C_m = 0.85 f_m' b a_b$

Where:

$$a_b = \left[\frac{e_{mu}}{e_{mu} + \frac{f_y}{E_s}}\right] d$$

$$= 0.85\left[\frac{0.003}{0.003 + 60/29000}\right] d$$

$$= 0.85(0.5918)d$$

$$= 0.503(140\text{inch})$$

$$= 70.43\text{inch}$$

Recall:

c = Distance to NA = $a_b/0.85$
= 70.428/0.85
= 82.86 in

$T = \Sigma A_s f_y$
$= 0.31 \text{ in}^2 (9.6 + 26.4 + 43.2 + 60)$ksi
$= 43.2$ kips

Now:

$C = \Sigma A_s f_y + 0.85 f_m' b a_b$
$= 0.31 \text{ in}^2(7.2 + 15.6 + 32.4 + 49.2 + 60 + 60)\text{ksi} + 0.85(1.5)(7.625)(70.428)$
$= 69.56 + 684.69$
$= 754.25$

Thus:

$P_b = C - T$
$= 754.25 - 43.2$
$= 711$ kips

6.Design balanced design axial strength (P_{bu})

$P_{bu} = \phi P_b$
$= 0.65 (711 \text{ kips})$
$= 462$ kips

7.Nominal balanced design moment strength (M_b): See Figure 11-29. Take sum of moments about plastic centroid (center of wall):

$M_b = A_s f_y - 0.85 f_m' a_b X_b b$
$= 0.31[60(68) + 43.2(52) + 26.4(36) + 9.6(20) - 7.2(4) + 15.6(4) + 32.4(20) + 49.2(36) + 52(60) + 68(60)] + 0.85(1.5)(70.428)(36.76)(7.625)$
$= 5308 + 25169$
$= 30477$ k-in
$= 2540$ k-ft

8. Design balanced design moment strength (M_{bu})

$M_{bu} = \phi M_b$
$= 0.65(2540 \text{ k-ft})$
$= 1651$ k-ft

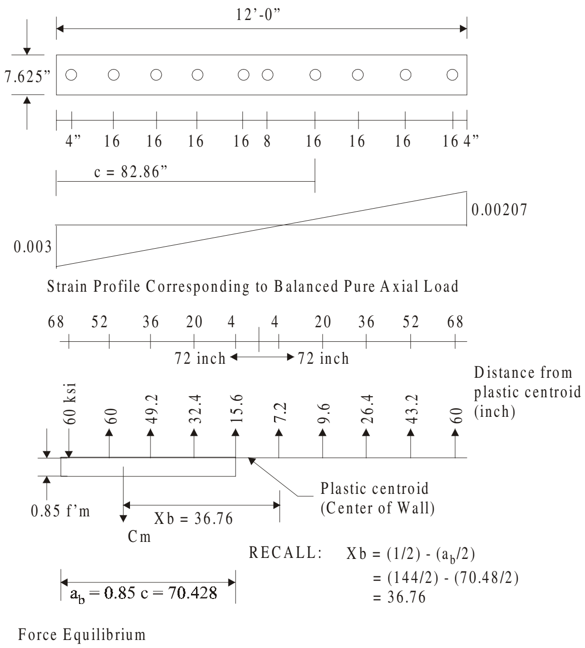

Figure 11-29. Balanced design load condition

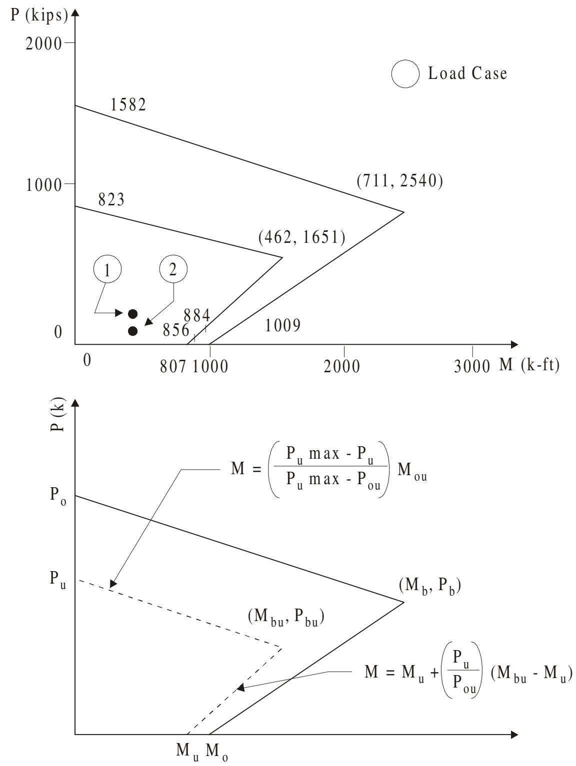

Figure 11-30. Interaction Diagram

B. Cracking moment strength

-Linearly elastic model
-Gross section properties

$(P/A) + M_{cr}/S = f_r$

Thus:

$M_{cr} = S[(P/A) + f_r]$

Where:

$A = bl = 7.625(144) = 1098 \text{ in}^2$
$s = bl^2/6 = 7.625\,(144)^2/6 = 26{,}352 \text{ in}^3$
$f_r = 4.0\sqrt{f_m'} = 4.0(1500)^{1/2} = 155 \text{ psi}$
P = Dead Load = 30,000 lbs

Thus:

Mcr = 26352[(30000/1098) + 155][(1/1000)(k/1b)]
= 4804.6 k-in
= 400 k-ft

C. Load Cases (See Figure 11-30)

Load Case 1:
$U = 12D + 1.0E$
$= 1.42D + 1.0E_h$

Therefore;
$U = 1.42(30) + 1.0(400)$
= 42.6 kips + 400 k-ft

From Figure 11-30: $P_u = 42.6 \text{kips} < P_{bu} = 462 \text{kips}$

Thus:

$P_{bu}/(M_{bu} - M_u) = P_u/M_x$
$M_x = (P_u/P_{bu})(M_{bu} - M_u)$
$M_n = M_u + M_x$
$= M_u + (P_u/P_{bu})(M_{bu} - M_u)$
= 807 k-ft + (42.6/462)(1651 - 807)
= 884.8 k-ft Nominal Flexural Moment Strength

Note: 884.8 k-ft > 400 k-ft OK

Note: M/M_{cr} = 884.8/400 = 2.2 > 1.8 OK (recall fully grouted wall)

Load Case 2:

U = 0.90D + 1.0E
U = 0.90(30) + 1.0(400)
= 27 kips + 400 k-ft

From Figure 11-30: P_u = 27 kips < P_{bu} = 462

Thus:
M_n = 807 + (27/462)(1651 - 807)
= 856 k-ft

Note:

M_n/M_{cr} = 856/400 = 2.14 > 1.8 ....OK

D. Boundary Elements

Section 2108.2.5.6 of the 1997 UBC states that:

"Boundary members shall be provided at the boundaries of shear walls when the compressive strains in the wall exceed 0.0015. The strain shall be determined using factored forces and R_w equal to 1.5"

Note that there is an error in the code since it refers to the obsolete R_w factor, which has been replaced by the R factor in the 1997 UBC. By comparing the values of the new R factor with the old R_w factor, one can conclude that the boundary member requirements should be calculated using an R of 1.1. Since the design forces for the bearing wall were calculated with an R factor of 4.5, the factored loads must be multiplied by 4.5/1.1 = 4.09 in order to determine if the moment capacity of the wall at a maximum compressive strain of 0.0015 is less than that required for boundary members.

To calculate the moment capacity at a maximum compressive strain of 0.0015, we can assume a linear compressive stress-strain relationship for the masonry. So, using a linear strain model, $f_m = 0.75f_m'$ for a strain of 0.0015: See figure 11-31.

_ Must solve for neutral axis (c)

_ Trial and error solution

_ Take moments about plastic centroid

Load Case 1:
U = 12D + 1.0E
= 1.42D + $1.0E_h$
(P_u = 42.6kips and M_u = 400 k-ft)

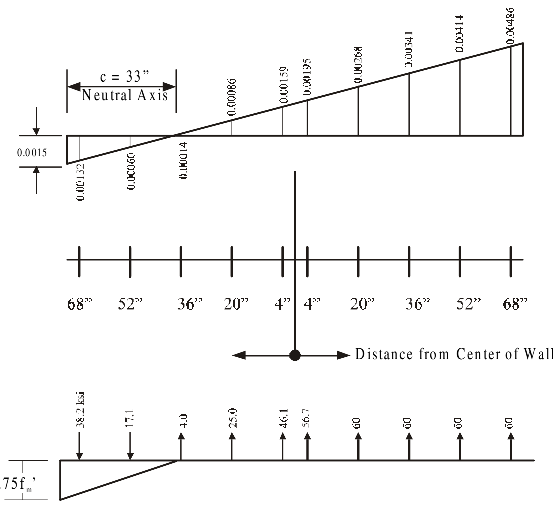

Figure 11-31. Stress/Strain relationship for determining boundary elements in masonry

By trial and error select depth to neutral axis, NA = 33.0 inches (See Figure 11-31 for stress and strain diagrams).

T = $A_s f_s$
= 0.31(4(60) + 56.7 + 46.1 + 25 +4)
= 115.3 kips
C = $A'_s f_s + 0.75f_m'$ cb/2
= 0.31(38.2 + 17.1)
+ 0.75(1.5)(33)(7.625)(1/2)
= 158.7 kips
C-T = 43.4 kips (P_u = 42.6 kips)…OK

Use: NA = 33.0 inches

Take moments about the center of the wall centroid to determine moment corresponding to $0.75f_m'$. If $4.09M_u$ is less than M_n confinement of vertical steel is not required.

M_n= $A_s f_s$(dist. to Center of Wall)
+ $0.75f'_m(c/2)(L/2-c/3)$

$= 0.31[68(38.2) + 52(17.1) - 36(4.0)$
$-20(25) - 4(46.1) + 4(56.7)$
$+60(20+36+52+ 68)]$
$+ 0.75(1.5)(33/2)[(144/2) - (33/3)]$
$= 441.7$ k-ft $< 4.09M_u = 1636$ k-ft

Thus,
Boundary Elements Required.

E. Shear

1. Shear Demand

Recall : $V_u > \phi V_n$
$V_u > \phi (V_m + V_s)$
$V_u = 1.0V_E$
$= 1.0(75$ kips$)$
$= 75$ kips

2. Shear strength with only CMU (no shear steel)

$V_n = V_m \ (V_s = 0)$
$= C_d A_{mv}(f'_m)^{1/2}$

Where:

$C_d \ \alpha \ M/Vd$
$d = 12$ ft $- (4/12)$ft $= 11.67$ ft
$V = 75$ kips
$M = 400$ k-ft
$M/Vd = 400/[75(11.67)] = 0.46$ (from Figure 10-26: $C_d = 2.06$)
$A_{mv} = l_w b = 144$ in$(7.625$ in$) = 1098$ in^2

Now:

$V_n = C_d A_{mv}\sqrt{f'_m}$; $C_d = 2.06$
$V_n = 2.06 \times 1098$ in$^2(1500$ psi$)^{1/2}/1000$ lb/k
$= 87.6$ kips
$V_u > \phi V_n$
$\phi V_n = 0.60(87.6$ kips$)$
$= 52.6$ kips
$V_u = 75 > 52.6$ ...NG shear reinforcement required

3. Design shear reinforcement to carry total shear (at least majority, authors preference)

$V_u = \phi V_n = \phi V_s$ ......$(V_m = 0)$
$V_u = A_{mv}\rho_n f_y \phi$

Recall:
$\rho_n = V_u/A_{mv}f_y\phi$
$= 75$ kips$/(1098$in$^2)(60$ k/in$^2)(0.60)$
$= 0.0019$
Now:
$A_v = 0.0019(12$ in$)(7.625$ in$)$
$= 0.174$ in^2/ft
USE: # 5 @ 16 in. o.c.
$(A_v = 0.23$ in^2/ft > 0.174 in^2/ft$)$

Thus, the steel can carry all the shear

4. Shear strength of steel only:

$$\phi V_s = \rho_n A_{mn} f_y \phi = \frac{0.23(1098\text{in}^2)(60\text{ksi})(0.60)}{(12\frac{\text{in}}{\text{ft}})(7.625\text{in})} = 99.36\text{kips}$$

5. Bottom (L_w) of wall
Shear strength of steel only with ϕ=0.85

$$V_s = 99.36\left(\frac{0.85}{0.60}\right)$$

$= 140.76$ kips > 75 kips OK

Table 11-10. Total Design Base Shear for 3-Story Building Wood Structural Panel Bearing Wall System

Notes	Item/Description	Total Design Base Shear (V) Seismic Zone and Factor				
		1 0.075	2A 0.15	2B 0.20	3 0.30	4 0.40
1	Cv	0.18	0.32	0.40	0.54	0.64Nv
	I	1.0	1.0	1.0	1.0	1.0
	R	5.5	5.5	5.5	5.5	5.5
2	T EQ. 10-10E	0.256	0.256	0.256	0.256	0.256
	Ca	0.12	0.22	0.28	0.36	0.44Na
3	Nv	-	-	-	-	1.2
3	Na	-	-	-	-	1.0
4	V EQ. 11-10A	0.128W	0.227W	0.284W	0.384W	0.545W
4	V EQ. 11-10B	0.055W*	0.10W*	0.127W*	0.164W*	0.20W*
4	V EQ. 11-10C	0.013W	0.024W	0.031W	0.039W	0.048W
4	V EQ. 11-10D	-	-	-	-	0.070W

Notes: 1. Soil profile type D
2. $T = C_t (h_n)3/4 = 0.256$ sec
For Ct = 0.020
h_n – 30 feet
3. Seismic source B
Closest distance to seismic source = 5km
4. * = Governs

Table 11-11. Total Design Base Shear for 3- Story Building Masonry Shear Wall Bearing Wall System

Notes	Item/Description	Total Design Base Shear (V) Seismic Zone and Factor				
		1 0.075	2A 0.15	2B 0.20	3 0.30	4 0.40
1	Cv	0.18	0.32	0.40	0.54	0.64Nv
	I	1.0	1.0	1.0	1.0	1.0
	R	4.5	4.5	4.5	4.5	4.5
2	T EQ. 10-10E	0.256	0.256	0.256	0.256	0.256
	Ca	0.12	0.22	0.28	0.36	0.44Na
3	Nv	-	-	-	-	1.2
3	Na	-	-	-	-	1.0
4	V EQ. 11-10A	0.156W	0.278W	0.347W	0.469W	0.67W
4	V EQ. 11-10B	0.067W	0.122W*	0.156W*	0.20W*	0.244W*
4	V EQ. 11-10C	0.013	0.024W	0.031W	0.039W	0.048W
4	V EQ. 11-10D	-	-	-	-	0.085W

Notes: 1. Soil profile type D
2. * = Governs